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The Differential Impacts of Critical Mineral Prices and Oil Prices on the Economy

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Abstract

This paper compares the impacts of critical mineral price and oil price on an economy in a unified neoclassical growth model. Unlike oil price shocks, which affect the cost of utilizing existing capital (e.g., cars), critical mineral price shocks influence the cost of creating new capital (e.g., electric vehicles) without altering the cost of existing capital. We find that both types of shocks ultimately reduce output and welfare. However, oil-price increases are systematically more contractionary for the economy. Mineral-price increases generate comparatively larger adjustments in investment, capital, and external borrowing but smaller and more gradual losses in output and welfare, and in capital-rich economies can slightly raise long-run employment. These results imply that oil-price shocks remain the more serious threat to aggregate activity and welfare, whereas mineral-price shocks call for policies that smooth investment and external-balance-sheet adjustment (e.g., macroprudential tools and precautionary reserves or fiscal buffers).

Keywords: Critical minerals; oil price shocks; commodity price shocks; energy transition.

JEL Codes: Q43; Q41; E32; F41; E22.

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1 Introduction

Over the past decades, economists have extensively studied how commodity price shocks propagate through economic systems, driven initially by the profound economic disruptions from oil price volatility. Seminal work by Hamilton (1983) established the significant role that oil price shocks played in triggering post-war recessions in the U.S., setting the stage for decades of empirical and theoretical exploration. Subsequent studies consistently confirmed the recessionary and inflationary effects of oil shocks, highlighting their roles in shaping business cycles and macroeconomic growth and stability (Kilian, 2009; Blanchard and Gali, 2007; Schubert and Turnovsky, 2011). However, as global economies increasingly pivot towards renewable energy and electrification, the focus has shifted to mineral-intensive technologies (Considine et al., 2023; Boer, Pescatori and Stuermer, 2021; Bogmans et al., 2024; Miranda-Pinto et al., 2024). In this paper, we explore how price shocks to critical minerals affect macroeconomic dynamics, contrasting these impacts explicitly with traditional oil price shocks within a unified neoclassical growth framework.

As global economies increasingly shift towards electrification and low-carbon energy, understanding the impact of critical mineral price shocks compared to traditional oil price shocks has become crucial. Critical minerals, such as lithium, cobalt, and rare earth elements, are more geographically concentrated than oil, with countries like China dominating global supply chains.[1] This large concentration exposes these markets greatly to geopolitical risks and trade disruptions, making metal prices potentially more volatile. The International Energy Agency (IEA) notes that many critical minerals experienced broad-based price increases in 2021 and early 2022, accompanied by strong volatility, particularly for nickel and lithium (International Energy Agency, 2023*b*). Moreover, the energy transition and rising demand for minerals may increase their share in GDP while the share of oil decreases. The IEA predicts that copper demand will rise by 50%, while oil consumption may fall by 25% by 2040 under a net-zero scenario (International Energy Agency, 2023*a*,*c*). These trends raise concerns that mineral price shocks could become increasingly important drivers of macroeconomic fluctuations, potentially surpassing oil in their economic impact.

We build on the standard neoclassical growth model (e.g., Solow (1956); Cass (1965); Koopmans (1963)) to explicitly contrast the macroeconomic impacts of critical mineral price shocks with those of traditional oil price shocks. The model embeds a representative household with consumption-leisure choice, a CES production with three factors (capital, labor, and oil), endogenous investment adjustment costs, and external borrowing subject to a country risk premium following the small open economy model in Turnovsky (2002); Schubert and Turnovsky (2011). We depart from the standard model by differentiating

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[1] For instance, China controls a significant share of the global supply chain for several critical minerals, including up to 80% of certain rare earth elements (International Energy Agency, 2023*b*).

how oil and minerals enter the production and investment processes. Oil is modeled as a flexible, variable input that affects the operational costs of existing capital. In contrast, critical minerals are introduced as an essential component of capital formation, influencing the marginal cost of investment and the installation of new capital, without affecting the productivity or cost structure of the existing capital stock.

This modeling distinction enables us to examine how these two types of shocks propagate differently through the economy and aligns with recent insights from International Energy Agency (2022) and Bogmans et al. (2024). The International Energy Agency (IEA) emphasizes that oil price shocks have immediate and broad effects on consumers, because oil is a fuel directly consumed in the daily operations of all gasoline- and diesel-powered vehicles. In contrast, critical minerals like copper or lithium are primarily embedded in investment goods such as batteries, new EVs, and renewable energy infrastructure—meaning that their supply disruptions primarily delay and raise the cost of future capital formation rather than affecting current usage. The 2024 World Economy Outlook by the International Monetary Fund (IMF) confirms that primary metals are deeply integrated into the production of investment goods, including machinery, vehicles, and electrical equipment.[2] Because of this capital-intensive role, metal price shocks tend to increase the cost of investment and propagate slowly but persistently through the economy. In contrast, oil and gas are mainly used as fuels in transport and utilities, leading to faster and more direct impacts on consumer prices and inflation.

We find that oil and mineral price shocks differ systematically in both their long-run effects and their transitional dynamics. Across all calibrations that keep the economy intertemporally viable, a permanent increase in the oil price is more contractionary for output, consumption, and welfare than a mineral-price increase of the same magnitude. Oil shocks induce persistent declines in oil use, capital, and activity, whereas mineral shocks generate sharper contractions in capital and stronger deleveraging but smaller and more gradual losses in output and welfare. These patterns are robust to variations in the elasticity of substitution and in the steepness of the borrowing-risk premium.

The mechanisms behind these results reflect the different roles of oil and minerals in the model. Oil enters as a variable input in production, so a higher oil price acts like an adverse technology shock that raises operating costs, depresses the marginal product of other factors, and reduces labor demand. Critical minerals are embodied in new capital and enter the convex installation cost and the cost of debt, so higher mineral prices raise the user cost of capital and erode net foreign wealth. This induces firms to cut investment and households to supply slightly more labor, leading to deeper falls in capital and debt but milder contractions in output and consumption, and in capital-rich economies, even small increases in long-run employment. Consequently, macro-

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[2] For example, metals represent more than 10 percent of direct input expenditure in US sectors for electrical equipment and machinery (Bogmans et al., 2024).

financial tools and production flexibility are more critical for managing mineral shocks than traditional demand-stabilization policies.

Our findings resonate closely with recent empirical and theoretical work that distinguishes metals from oil as macroeconomic drivers. Miranda-Pinto et al. (2024) show, in a production-network model disciplined by local-projection evidence, that metals price shocks—because metals are key inputs into investment goods such as machinery, electrical equipment, and construction—generate gradual but persistent effects on both core and headline inflation, whereas oil supply shocks mainly move headline inflation and are more short-lived. This persistence on the price side mirrors our mechanism on the quantity side, where mineral shocks propagate slowly through capital formation and wealth balance sheets and are less acutely contractionary for output and welfare than oil shocks of the same size. Likewise, Considine et al. (2023) employ a global vector autoregression framework to analyze the spillover effects of critical mineral price shocks and find that these shocks are beginning to affect macroeconomic indicators such as inflation, albeit less dramatically than oil price shocks.

Our analysis extends a broad literature examining how commodity price shocks ripple through economies. Early foundational work showed that oil shocks induce stagflation and output declines (Hamilton, 1983, 2003), and later studies demonstrated that the severity of these effects depends critically on production structures and substitution elasticities (Kilian, 2009; Schubert and Turnovsky, 2011; Kilian and Murphy, 2014). Although extensive research has documented the disruptive consequences of oil price volatility on macroeconomic stability, relatively little attention has been directed toward metal price shocks. By complementing recent exceptions (Considine et al., 2023; Miranda-Pinto et al., 2024), we provide the first unified theoretical framework that contrasts oil and critical mineral shocks. Specifically, we adapt a canonical one-sector open-economy neoclassical growth model to incorporate both oil's contemporaneous operating-cost channel and minerals' intertemporal investment-cost channel. In doing so, we offer a complete analytical characterization of the long-run equilibrium responses—showing how these hinge on input shares, substitution elasticities, and financial constraints—while also revealing richer transitional dynamics through numerical simulations.

The remainder of the paper is structured as follows: Section 2 presents the neoclassical growth model with critical mineral and oil inputs, identifying the mechanisms through which each shock propagates. Section 3 characterizes the steady-state and transitional dynamics analytically and numerically, providing comparative elasticities and simulated results. Section 4 concludes.

2 Model

2.1 Environment and equilibrium

We extend the standard textbook one-sector neoclassical growth model of an open economy to compare the impacts of oil prices versus critical mineral prices on the economy. We also build on the models in Turnovsky (2002) and Schubert and Turnovsky (2011) to consider a small open economy inhabited by an infinitely-lived representative household that chooses labor supply, consumption, investment, foreign borrowing, and imports of oil (with trade balanced in every period). The key departure from the canonical neoclassical framework is that we also consider imports of critical minerals, and whereas oil enters production as a *variable* input each period, minerals enter only through the *fixed* cost of installing new capital. This structure permits a clean comparison of shocks that work through current operating costs versus those that act through future capacity.

Preferences and labor allocation The household is endowed with one unit of time per capita. Let l_t denote leisure and $L_t \equiv 1 - l_t$ labor. Lifetime welfare is

$$U_0 = \int_0^\infty e^{-\beta t} \frac{\left[C_t l_t^\theta\right]^\gamma}{\gamma} dt, \qquad \beta > 0,\ \theta > 0,\ \gamma < 1, \tag{1}$$

so the marginal value of consumption is higher when leisure is plentiful and the elasticity of intertemporal substitution equals $1/(1-\gamma)$.

Production technology Output of the traded good combines capital K_t, labor L_t and imported oil Z_t through a three-input CES aggregator

$$Y_t \ = \ F(K_t, L_t, Z_t) \ = \ A\left[\alpha_K K_t^{-\rho} + \alpha_L L_t^{-\rho} + \alpha_Z Z_t^{-\rho}\right]^{-1/\rho}, \qquad \alpha_K + \alpha_L + \alpha_Z = 1, \tag{2}$$

where $A > 0$ measures Hicks-neutral productivity and $\rho > -1$ governs curvature. The elasticity of substitution among the three inputs is $\sigma \equiv 1/(1+\rho)$: when $\rho \to 0$ the function becomes Cobb-Douglas; as $\rho \to \infty$ inputs turn Leontief.

Investment and critical minerals Building new capital requires critical minerals purchased abroad at the exogenous world price w_t. Capital accumulation obeys

$$\dot{K}_t = mI_t - (n+\delta)K_t, \quad 1 \geq m > 0, n > 0,\ \delta > 0, \tag{3}$$

where I_t is investment of critical minerals, m converts the efficiency of critical mineral units into productive capital, n is the population growth, and δ is the depreciation rate. Installing new capital of course incurs the convex cost:

$$\Phi(I_t, K_t) = mw_tI_t + \tfrac{h}{2}\, I_t^2/K_t, \qquad h > 0. \tag{4}$$

A higher mineral price, therefore, shifts marginal installation costs upward without changing their curvature, reflecting the straight pass-through from battery-grade minerals to the sticker prices of new capital goods (EVs).

External borrowing and risk premium Following Schubert and Turnovsky (2011), we assume that foreigh lenders view the country's debt-to-capital ratio as a sufficient statistic for default risk. Let $B_t > 0$ denote debt and q_t the shadow value of capital in bond units; then

$$r_t = r_f + \exp\left(\eta \times \frac{B_t}{q_t K_t}\right) - 1, \quad \eta > 0, \tag{5}$$

where r_f is the world risk-free rate. A higher debt burden therefore raises the cost of funds and feeds back into real decisions.[3] .

Intertemporal budget constraint Let the output good be numeraire and p_t denote the world oil price. The per–capita flow budget constraint is

$$\dot{B}_t = (r_t - n)B_t + C_t + p_t Z_t + \Phi(I_t, K_t) - F(K_t, L_t, Z_t). \tag{6}$$

This budget constraint clearly highlights the difference between the oil price and the critical mineral price. Oil shocks hit immediately through contemporaneous operating cost $p_t Z_t$; mineral shocks propagate intertemporally by raising $\Phi(I_t, K_t)$, damping investment, lowering future capital.

Equilibrium allocation Taking $\{p_t, w_t, r_f\}$ and the risk-premium schedule (5) as given, the representative household chooses $(C_t, l_t, L_t, Z_t, I_t, B_t, K_t)_{t \geq 0}$ to maximise (1) subject to (2)-(6). Let λ_t be the co-state variable on B_t and q_t the co-state on K_t (both in units of the traded good). The optimality conditions are

$$C_t^{\gamma-1} l_t^{\theta\gamma} = \lambda_t, \tag{7}$$

$$\frac{\theta C_t}{l_t} = F_L(K_t, L_t, Z_t), \tag{8}$$

$$F_Z(K_t, L_t, Z_t) = p_t, \tag{9}$$

$$\frac{1}{m}\left(m w_t + \frac{h I_t}{K_t}\right) = q_t, \tag{10}$$

$$\beta - \frac{\dot{\lambda}_t}{\lambda_t} = r_t - n, \tag{11}$$

$$\frac{F_K(K_t, L_t, Z_t)}{q_t} + \frac{\dot{q}_t}{q_t} + \frac{h I_t^2}{2 q_t K_t^2} - \delta = r_t, \tag{12}$$

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[3] The representative agent, in making his individual decisions, treats the interest rate as fixed. The reason is that the interest rate facing the debtor nation depends on the economy's aggregate debt, which the individual agent, being atomistic, rationally assumes is beyond his control; see Schubert and Turnovsky (2011); Turnovsky (2002).

together with the transversality conditions

$$\lim_{t\to\infty} e^{-\beta t}\lambda_t B_t = 0, \qquad \lim_{t\to\infty} e^{-\beta t}\lambda_t q_t K_t = 0.$$

The co-state λ_t is the marginal value of net foreign assets in the form of internationally traded bonds. Equation (7) equates the marginal utility of consumption to this shadow value, while (11) is the Euler condition linking its evolution to the gap between the borrowing rate $r_t - n$ and the rate of time preference β. Condition (8) is the intratemporal labor-leisure trade-off: the marginal rate of substitution between leisure and consumption equals the marginal product of labor. Equation (9) says that oil is demanded up to the point where its marginal product equals the world price.

The variable q_t is the value of capital in terms of the (unitary) price of foreign bonds. Condition (10) sets this value equal to the marginal installation cost of capital, which depends on the mineral price w_t and adjustment costs. Finally, (12) is an asset-pricing condition: the return to holding domestic capital (dividends F_K/q_t, capital gains $\dot{q}_t/q_t$, and the saving in future installation costs net of depreciation) must match the return on foreign bonds r_t. Mineral prices thus enter the intertemporal margin through the cost of investment and cost of capital q_t, whereas oil prices act through the static input choice in (9).

Combining the optimality conditions with the capital accumulation equations and the flow budget constraint yields a four-dimensional autonomous system in per-capita capital, external debt, the shadow value of capital, and leisure, (K_t, B_t, q_t, l_t). This system is nonlinear and does not admit a closed-form solution (see Appendix A for derivation). In Section 3.2, we study the local dynamics around the steady state by numerically solving the linearized system for alternative oil and mineral price shocks.

2.2 Steady-state equilibrium

The steady state of the economy (denoted by tildes) is reached when $\dot{K} = \dot{B} = \dot{q} = \dot{l} = \dot{\lambda} = 0$. It is then determined by

$$\tilde{q} = w + \frac{h(n+\delta)}{m^2}, \tag{13a}$$

$$\tilde{r} = r\left(\frac{\tilde{B}}{\tilde{q}\,\tilde{K}}\right) = \beta + n, \tag{13b}$$

$$\tilde{l} = 1 - \tilde{L}, \tag{13c}$$

$$\tilde{Y} = F(\tilde{K}, \tilde{L}, \tilde{Z}), \tag{13d}$$

$$\frac{\theta\tilde{C}}{\tilde{l}} = F_L(\tilde{K}, \tilde{L}, \tilde{Z}), \tag{13e}$$

$$F_Z(\tilde{K}, \tilde{L}, \tilde{Z}) = p, \tag{13f}$$

$$\frac{F_K(\tilde{K}, \tilde{L}, \tilde{Z})}{\tilde{q}} + \frac{m^2(\tilde{q} - w)^2}{2h\,\tilde{q}} = \beta + n + \delta, \tag{13g}$$

$$\beta\tilde{B} + \tilde{C} + p\tilde{Z} + \frac{m^2(\tilde{q}^2 - w^2)}{2h}\,\tilde{K} = F(\tilde{K}, \tilde{L}, \tilde{Z}), \tag{13h}$$

$$\tilde{I} = \frac{n+\delta}{m}\,\tilde{K}. \tag{13i}$$

These nine equations pin down the steady-state values $(\tilde{K}, \tilde{L}, \tilde{l}, \tilde{Z}, \tilde{Y}, \tilde{C}, \tilde{B}, \tilde{q}, \tilde{I})$ for given prices and parameters. The recursive structure implies that the long-run price of installed capital $\tilde{q}$, the steady-state borrowing rate, and the debt-equity ratio $\tilde{B}/(\tilde{q}\,\tilde{K})$ do not depend on the oil price p. In contrast, mineral prices enter directly through (13a) and (13g), so changes in mineral prices affect the steady-state user cost of capital and hence the capital intensity of the economy.

3 The impacts of critical mineral price versus oil price

3.1 Long-run effects: closed-form elasticities

Remark 1. *If the economy is intertemporally viable in the sense of having net positive wealth, so that $\tilde{q}\tilde{K} > \tilde{B}$, then the elasticities of output, capital, oil, and consumption with respect to the oil price are negative, i.e. $\epsilon_Y < 0$, $\epsilon_K < 0$, $\epsilon_Z < 0$, and $\epsilon_C < 0$.*

Proof. Following Schubert and Turnovsky (2011). □

Remark 2. *If $\tilde{q}\tilde{K} > \tilde{B}$, $\sigma \geq 1$, and $\frac{s_K}{s_Z} > \frac{\sigma}{D}$, then $\varepsilon_Y > \epsilon_Y$ and $\varepsilon_L > \epsilon_L$, where ε denotes long-run elasticities with respect to permanent changes in the critical mineral price, and*

Table 1: Long-run effects of an increase in oil price and critical mineral price

Impact on	Elasticity w.r.t. oil price	Elasticity w.r.t. critical mineral price
Output	$\epsilon_Y = [L(1+\theta)(1-\sigma)-1]\,\dfrac{s_Z}{s_L}$	$\varepsilon_Y = \dfrac{s_K}{s_L}\dfrac{D}{\sigma}\,[L(1+\theta)(1-\sigma)-1] + E = \dfrac{s_K}{s_Z}\dfrac{D}{\sigma}\,\epsilon_Y + E$
Labor	$\epsilon_L = \epsilon_Y + \sigma\dfrac{s_Z}{s_L}$	$\varepsilon_L = \varepsilon_Y + D\dfrac{s_K}{s_L}$
	$\epsilon_L = [L(1+\theta)-1]\,(1-\sigma)\dfrac{s_Z}{s_L}$	$\varepsilon_L = \dfrac{s_K}{s_L}\dfrac{D}{\sigma}(1-\sigma)\,[L(1+\theta)-1] + E = \dfrac{s_K}{s_Z}\dfrac{D}{\sigma}\,\epsilon_L + E$
Capital	$\epsilon_K = \epsilon_Y$	$\varepsilon_K = \varepsilon_Y - D$
Oil	$\epsilon_Z = \epsilon_Y - \sigma$	$\varepsilon_Z = \varepsilon_Y$
Consumption	$\epsilon_C = \epsilon_Y - \dfrac{pZ(1-\sigma)}{C} = \epsilon_Y - \dfrac{\theta L(1-\sigma)}{l}\dfrac{s_Z}{s_L}$	$\varepsilon_C = \varepsilon_Y - \dfrac{\theta L(1-\sigma)}{l}\dfrac{D}{\sigma}\dfrac{s_K}{s_L} - \dfrac{E}{l}$
	$\epsilon_C = -\left[1+\dfrac{L}{l}\big(\theta - l(1+\theta)\big)(1-\sigma)\right]\dfrac{s_Z}{s_L}$	$\varepsilon_C = -\dfrac{s_K}{s_L}\dfrac{D}{\sigma}\left[1+\dfrac{L}{l}\big(\theta - l(1+\theta)\big)(1-\sigma)\right] - \dfrac{EL}{l}$
		$\varepsilon_C = \dfrac{s_K}{s_Z}\dfrac{D}{\sigma}\,\epsilon_C - \dfrac{EL}{l}$

Note: $D \equiv \dfrac{w(\beta+n+\delta)\sigma}{q(\beta+n+\delta) - \dfrac{h(n+\delta)^2}{2m^2}} < \sigma$ and $E \equiv \dfrac{s_K}{s_L}\dfrac{L\theta}{qKF_K}\beta(qK-B)(qD-w) = \dfrac{\beta l}{C}(qK - B)\left(D - \dfrac{w}{q}\right)$. For $\sigma \geq 1$ and $qK > B$ we have $E > 0$. All values here are steady-state values and we drop the tilde notation for convenience.

ϵ denotes long-run elasticities with respect to permanent changes . This implies that in economies where the capital share is sufficiently large relative to the oil share, the long-run impacts of a positive mineral-price shock on output and labor are less severe than those of an oil-price shock of the same magnitude.

Proof. Note that

$$\frac{h(n+\delta)^2}{2m^2} = q(\beta+n+\delta) - F_K = \frac{qDF_K}{w\sigma} - F_K = F_K\left(\frac{qD}{w\sigma} - 1\right),$$

which implies $\frac{qD}{w\sigma} - 1 > 0$. If $\sigma \geq 1$, then $\frac{qD}{w} - 1 \geq \frac{qD}{w\sigma} - 1 > 0$, so $qD > w$ and hence $E > 0$. Using $\varepsilon_Y = \dfrac{s_K}{s_Z}\dfrac{D}{\sigma}\epsilon_Y + E$, a sufficiently large capital-oil share ratio $\dfrac{s_K}{s_Z} > \dfrac{\sigma}{D}$ ensures that ε_Y is less negative than ϵ_Y, and similarly for ε_L relative to ϵ_L. $\square$

Table 1 summarizes the long-run elasticities of the main macroeconomic aggregates with respect to permanent changes in the oil price (denoted by ϵ), and with respect to permanent changes in the critical mineral price (denoted by ε). For oil, the key feature is that the user cost of capital and the debt-equity ratio are independent of the oil price in the steady state. As a result, all long-run oil-price elasticities in Table 1 are governed purely by internal production conditions (factor shares s_K, s_L, s_Z and the elasticity of substitution σ) and by labor-leisure preferences (L, l, θ). In particular, we have $\epsilon_Y = \epsilon_K$, $\epsilon_Z = \epsilon_Y - \sigma$, and $\epsilon_L = \epsilon_Y + \sigma\, s_Z/s_L$, so that the capital-oil ratio rises by σ while the capital-labor ratio falls by $\sigma s_Z/s_L$. Under the intertemporal viability condition $qK > B$, Remark 1 shows that the long-run elasticities of output, capital, oil use and consumption with respect to the oil price are all negative. The magnitude of these declines depend on the elasticity of substitution: when $\sigma = 1$ (Cobb-Douglas), employment is unchanged, while output, capital, and consumption all fall in proportion to the oil-labor share ratio, and oil usage falls even more. As the technology departs from Cobb-Douglas ($\sigma \neq 1$), the degree of substitutability between inputs becomes crucial, and this directly shapes how sharply the economy can reallocate away from more expensive oil.

By contrast, mineral price shocks operate not only through technology, but also through the external-finance block of the model. This is reflected in the appearance of the two new objects D and E in the right-hand column of Table 1. The factor

$$D \equiv \frac{w(\beta + n + \delta)\sigma}{q(\beta + n + \delta) - \frac{h(n+\delta)^2}{2m^2}} = \sigma \, \frac{w(\beta + n + \delta)}{F_K}$$

plays the role of a *finance-attenuated elasticity of substitution*: it measures how strongly a change in the mineral price passes through to the user cost of capital. In the frictionless benchmark ($q \approx w$, $h/m^2 \to 0$), D is close to σ; with tighter borrowing conditions, D falls below σ, implying that the direct substitution channel from a mineral-price shock is damped by financial frictions. The term

$$E \equiv \frac{s_K}{s_L} \frac{L\theta}{qKF_K} \beta(qK - B)(qD - w) = \frac{\beta l}{C}(qK - B)\left(D - \frac{w}{q}\right)$$

captures a wealth/balance-sheet channel that does not appear in the oil-price elasticities. When the economy is a net creditor and inputs are at least Cobb-Douglas substitutes ($\sigma \geq 1$), we have $E > 0$: a higher mineral price tightens external finance, erodes wealth, and induces households to work more and reduce consumption in order to rebuild their balance sheet. In this sense, D summarizes how mineral prices pass through to the user cost of capital given financial structure, while E summarizes how the resulting wealth effects feed back into labor supply and consumption.

Given these mechanisms, the long-run impact of a mineral price increase depends on two sets of factors. First, all the "technological" determinants that matter for the oil-price elasticities (factor shares and σ) enter again, but scaled by the relative importance of capital and oil and by the pass-through factor $(s_K/s_Z)(D/\sigma)$. Second, the structure of external finance and the net foreign asset position determine the magnitude and sign of E, and thus the strength of the wealth channel. The Cobb-Douglas case is again particularly transparent. When $\sigma = 1$, we obtain $\varepsilon_Y = \varepsilon_Z = -Ds_K/s_L + E$, $\varepsilon_L = E$, and $\varepsilon_C = -Ds_K/s_L - (EL)/l$. This means that under positive net wealth, a mineral-price increase can raise long-run employment even though it still depresses output and consumption through the substitution channel.

Finally, we can compare the two price shocks more systematically. An oil-price increase works as a pure technology shock: its long-run impact is driven by internal production conditions and preferences and, under intertemporal viability, it unambiguously lowers output, capital, oil use and consumption (Remark 1). A mineral-price increase combines a weaker direct substitution effect with an additional wealth channel as we discuss above. When the capital share is large relative to the oil share, this difference becomes quantitatively important. Remark 2 shows that, in economies with net positive wealth and sufficiently high capital-oil share ratio, a permanent mineral-price increase is *less contractionary* for output and employment than an oil-price increase of the same size, and long-run employment may even rise slightly. An additional diagnostic implication is that the capital-oil ratio rises following an adverse oil-price shock (because firms substitute towards capital), but falls following an adverse mineral-price shock (because the user cost of capital is directly hit by tighter external finance). Overall, while both types of shocks are ultimately governed by the elasticity of substitution, only mineral-price shocks make the long-run macroeconomic adjustment depend critically on access to external financial markets.

3.2 Numerical simulations for transitional dynamics

To quantify the transitional effects of oil and critical mineral price shocks, we solve the saddle-path stable dynamics of the four-dimensional system in (K_t, B_t, q_t, l_t) around the steady state.[4] Closed-form solutions are not available, so we study a linear approximation of this system and compute transition paths numerically for alternative price experiments. Normalizing initial population $N_0 = 1$, lifetime welfare is $W = \frac{1}{\gamma}\int_0^\infty [C(t)\,l(t)^\theta]^\gamma e^{-\beta t}\,dt$, which in steady state simplifies to $W = \frac{1}{\gamma}\frac{\tilde{C}^\gamma \tilde{l}^{\theta\gamma}}{\beta}$.

Table 2: Benchmark parameter values

Preferences	
Risk aversion and discount rate	$\gamma = -1.5, \quad \beta = 0.04$
Utility weight on leisure	$\theta = 1.75$
Technology and demographics	
TFP and depreciation	$A = 1, \quad \delta = 0.05$
Factor income shares	$\alpha_K = 0.35, \quad \alpha_L = 0.60, \quad \alpha_Z = 0.05$
Elasticity of substitution (OECD)	$\sigma = 1.25$ $(\rho = -0.2)$
Elasticity of substitution (developing)	$\sigma = 0.9$ $(\rho = 1/9)$
Population growth	$n = 0.015$
Capital adjustment and minerals	
Mineral intensity of investment (benchmark)	$m = 0.14$ (minerals $\approx 4.9\%$ of GDP)
Capital adjustment cost	$h = 12$
International finance and prices	
World risk-free interest rate	$r_f = 0.045$
Debt-premium parameter	$\eta = 0.1$
Initial oil price	$p_0 = 1$
Initial mineral price	$w_0 = 1$
Shock sizes (same-percent scheme)	$p: 1 \to 2, \quad w: 1 \to 2$

We calibrate the benchmark economy using the parameter values in Table 2, which follow Schubert and Turnovsky (2011) for the oil block and extend their calibration to the

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[4] The derivation of the dynamic system is provided in Appendix A.

mineral intensity of investment. The preference parameters (γ, β, θ) are chosen so that the implied intertemporal elasticity of substitution and elasticity of leisure lie in standard macroeconomic ranges. On the production side, we set $(\alpha_K, \alpha_L, \alpha_Z) = (0.35, 0.60, 0.05)$, interpreting α_Z as the long-run share of oil in GDP for a representative net oil-importing economy. As discussed in Schubert and Turnovsky (2011), values around 4–5% are consistent with OECD and IEA evidence on energy/oil expenditure shares in GDP (e.g. Segal, 2007; International Energy Agency, 2024), so $\alpha_Z = 0.05$ provides a convenient benchmark. The mineral intensity parameter m is chosen so that the implied minerals share in GDP is slightly below the oil share. With a 35% capital share, the benchmark $m = 0.14$ implies a mineral share around 4.9% of GDP.

The elasticity of substitution σ is calibrated to capture systematic differences in production flexibility between OECD and developing economies. In a CES technology, $\sigma > 1$ indicates relatively easy substitution across inputs, whereas $\sigma < 1$ implies stronger complementarity. Cross-country estimates such as Duffy and Papageorgiou (2000) suggest that poorer economies tend to have elasticities below unity (around 0.8 or higher), while richer economies tend to exhibit elasticities slightly above one. Guided by this evidence and by Schubert and Turnovsky (2011), we take $\sigma = 0.9$ as the benchmark for developing economies and $\sigma = 1.25$ for OECD-type economies, and later vary σ over a modest range in our sensitivity analysis.

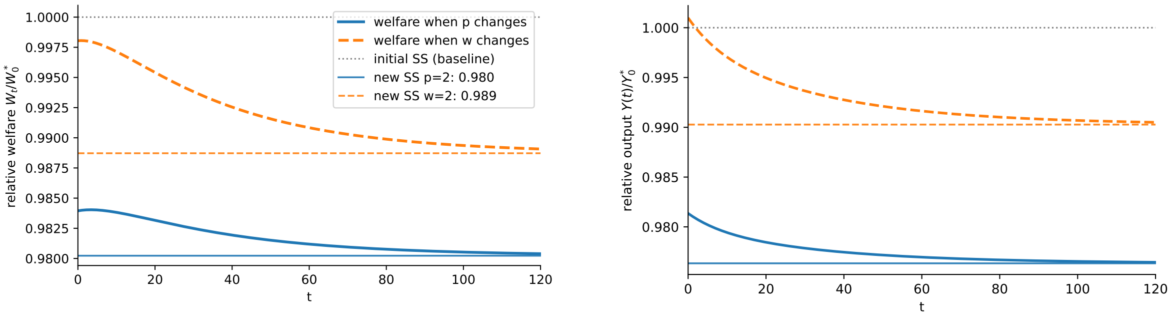
The remaining parameters govern the strength of capital adjustment costs and access to world financial markets. A convex installation cost parameter $h = 12$ and risk-free rate $r_f = 0.045$ are inherited directly from Schubert and Turnovsky (2011). Access to external finance is summarized by the debt-premium parameter η, with benchmark $\eta = 0.1$ and alternative values $\eta \in \{0.01, 1\}$ to span cases from near-frictionless borrowing to very tight credit markets.

Starting from the initial steady state with $p_0 = w_0 = 1$, we characterize the transitional dynamics under two simulation designs. Under the *same-percent-shock* scheme, we compare the transition when oil and mineral prices each permanently double from 1 to 2. Under the *same-budget-debt* scheme, we instead choose the new mineral price so that the steady-state debt matches the level attained when the oil price doubles. In subsequent subsections, we use this calibration and systematic variations in (σ, η) to document how transitional paths for the economy outcomes differ across oil and mineral shocks, and how they depend on production flexibility and financial frictions.

3.2.1 Benchmark adjustment to oil and mineral price increases

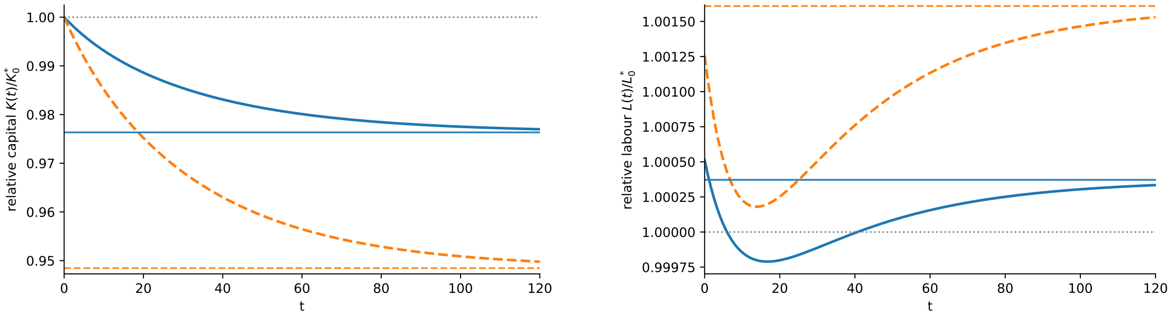
Figure 1 reports the transition paths for the OECD calibration ($\sigma = 1.25,\ \eta = 0.1$) when either the oil price doubles ($p : 1 \to 2$, solid blue) or the mineral price doubles ($w : 1 \to 2$, dashed orange). Each panel shows a variable normalised by its initial steady state: welfare

Figure 1: Benchmark adjustment to oil and mineral price doubles (OECD calibration)



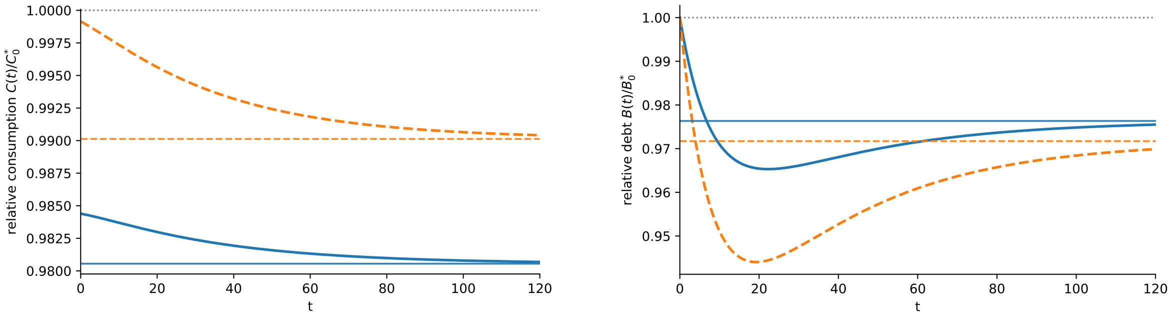
(a) Welfare $W(t)/W_0$

(b) Output $Y(t)/Y_0$



(c) Capital $K(t)/K_0$

(d) Employment $L(t)/L_0$



(e) Consumption $C(t)/C_0$

(f) External debt $B(t)/B_0$

Notes: OECD benchmark with $\sigma = 1.25$, $\eta = 0.1$, $m = 0.14$. All variables are shown relative to their initial steady-state values (pre-shock level 1.0). The solid line reports the response to an oil price doubling ($p : 1 \to 2$) and the dashed line reports the response to a mineral price doubling ($w : 1 \to 2$).

$W(t)/W_0$, output $Y(t)/Y_0$, capital $K(t)/K_0$, employment $L(t)/L_0$, consumption $C(t)/C_0$, and external debt $B(t)/B_0$. Horizontal reference lines refer to the new steady–state levels corresponding to each type of shocks. Table 3 reports numerical changes in percentage for a wider range of parameters.

The welfare, output, and consumption panels highlight that, even in the short run, an oil-price spike is more damaging than an equally sized mineral-price increase. Instantaneous welfare drops immediately and visibly more after the oil shock than after the mineral shock, and converges to a lower long-run level that is in line with the larger absolute welfare elasticity with respect to oil price than to mineral price in Table 1. Output falls on impact by nearly 2% when the oil price doubles and then declines monotonically toward a new steady state roughly 2.4% below the initial level. By contrast, a mineral-price doubling leaves output essentially unchanged on impact (a small positive blip in Table 3) and only gradually drifts down, to a long-run loss of about 1%. Consumption closely tracks output in both cases, but the decline is again about twice as large under the oil shock as under the mineral shock.

The capital and debt panels reveal that the milder output and consumption response to mineral-price shocks comes at the cost of a much larger contraction in the capital stock and a stronger balance-sheet adjustment. Because capital is predetermined, both paths start from $K(0)/K_0 = 1$. Thereafter, the oil shock induces a gradual decline in $K(t)$ toward a new steady state roughly 2.4% below the initial level, whereas the mineral shock generates a steeper and deeper fall, with the capital stock eventually contracting by about 5%. This is exactly what the long-run elasticities in Table 1 imply: the mineral price enters the user cost of capital; so a higher mineral price depresses investment and capital accumulation more strongly than a higher oil price, even though the resulting output loss is smaller. External debt $B(t)$ also falls in both cases as the economy improves its net foreign asset position, but the decline is more pronounced under the mineral shock.

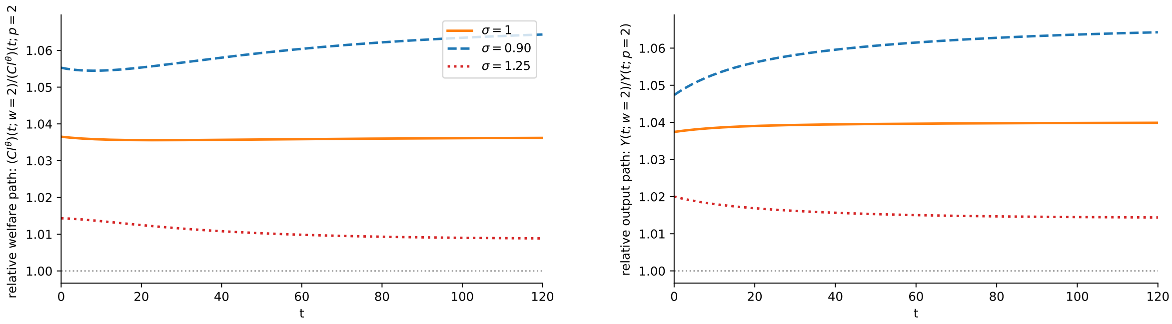
Labor dynamics underscore a key qualitative difference between the two shocks. In line with the long-run elasticity result that employment is essentially pinned down in the Cobb–Douglas case and only slightly perturbed when $\sigma \neq 1$, the deviations of $L(t)/L_0$ from unity are small throughout the transition. Nevertheless, the sign of the response differs systematically. After an oil-price increase, employment initially dips below its baseline, reflecting the contractionary effect of higher operating costs on labor demand, and only slowly returns close to its original level. By contrast, after a mineral-price increase, employment rises above baseline and remains slightly higher in the long run. This configuration is exactly what the mineral-price long-run elasticities and Remark 2 predict: with $\sigma > 1$, firms optimally substitute labor for now more expensive capital, while households, facing lower wealth and tighter external finance, supply more labor.

Taken together, the benchmark simulation confirms the analytical message of Section 3.1. The oil price behaves like a pure technology shock: It raises the cost of oil without changing the long-run user cost of capital, so the main adjustment is a persistent fall in oil use, capital, and output, with only modest movements in labor and debt. By contrast, the mineral price hits both the technology block and the financial block. It generates a sharper contraction in capital and external debt but a noticeably smaller recession in output and welfare. For a capital-rich OECD-type economy, this implies that unexpected oil-price spikes are more damaging for aggregate welfare than equally sized mineral-price shocks, even though mineral shocks have a stronger impact on the capital stock and on the cost of external finance.

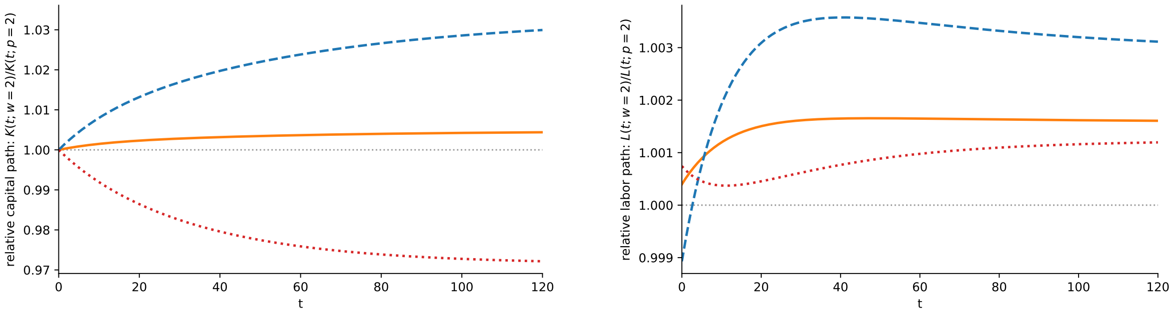
3.2.2 Sensitivity to the elasticity of substitution σ

Figure 2: Sensitivity to elasticity of substitution: mineral vs oil price doubles

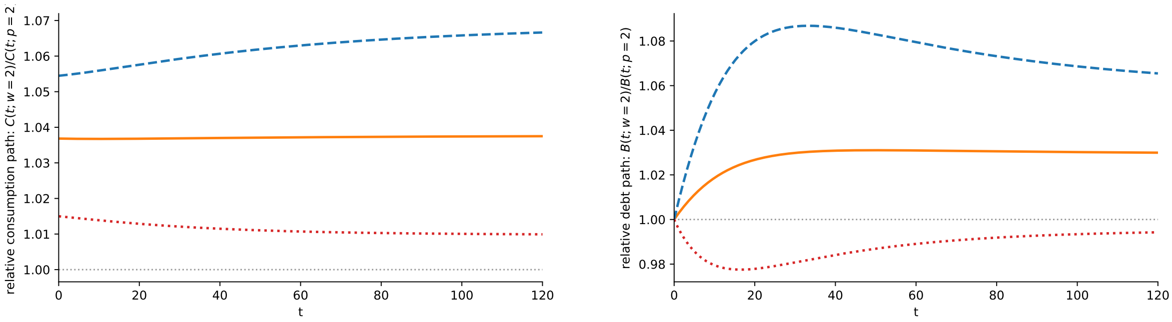
(a) Welfare ratio $W_w(t)/W_p(t)$ (b) Output ratio $Y_w(t)/Y_p(t)$



(c) Capital ratio $K_w(t)/K_p(t)$ (d) Employment ratio $L_w(t)/L_p(t)$



(e) Consumption ratio $C_w(t)/C_p(t)$ (f) Debt ratio $B_w(t)/B_p(t)$



Notes: OECD benchmark with $\eta = 0.1$ and $m = 0.14$. Each panel plots the ratio of the transitional path after a mineral price doubling ($w : 1 \to 2$) to the path after an oil price doubling ($p : 1 \to 2$). Curves correspond to three elasticities of substitution: $\sigma = 1$ (Cobb-Douglas), $\sigma = 0.9$, and $\sigma = 1.25$ (see legend).

Figure 2 summarizes how the relative transitional effects of oil and mineral price shocks depend on production flexibility. Each panel plots the ratio of the path after a mineral price doubling to the path after an oil price doubling for a given variable $X \in \{W, Y, K, L, C, B\}$, i.e. $X_w(t)/X_p(t)$ with $(w : 1 \to 2)$ and $(p : 1 \to 2)$. Curves correspond to three values of the elasticity of substitution, $\sigma \in \{0.9, 1, 1.25\}$, holding all other parameters at their benchmark levels. The blue dashed line refers to $\sigma = 0.9$, the orange solid line to $\sigma = 1$, and the red dotted line to $\sigma = 1.25$. A value above one means that the mineral shock is less contractionary for that variable than the oil shock at that horizon, whereas a value below one implies the opposite.

The welfare, output, and consumption panels convey a clear message: For all three elasticities, mineral price hikes generate milder recessions than oil price hikes. The ratios $W_w(t)/W_p(t)$, $Y_w(t)/Y_p(t)$, and $C_w(t)/C_p(t)$ lie above one along the entire transition and converge to long-run values greater than one. The gap is largest when $\sigma = 0.9$, reflecting that in more inflexible technologies inputs are relatively complementary, so firms cannot easily substitute away from expensive oil; the output and welfare losses from an oil shock then grow large relative to those from a mineral shock. As σ rises toward 1.25, the ratio paths flatten and move closer to one: Greater substitutability dampens the impact of both shocks, but it attenuates the oil-price elasticities more strongly than the mineral-price elasticities.

The capital and debt panels highlight a richer interaction between substitution possibilities and the user-cost and balance-sheet channels. When σ is below or close to one, the capital ratio $K_w(t)/K_p(t)$ stays above one along the transition and converges to a value slightly greater than one: Capital falls more after an oil-price doubling than after a mineral-price doubling. In this relatively inflexible region, the dominant effect of an oil shock is to cut back energy use and thereby reduce the marginal product of capital, even though the user cost of capital is unchanged. As σ rises to 1.25, the picture reverses: Firms can more easily substitute away from oil, so the contractionary effect of the oil shock on capital weakens, whereas the mineral shock works mainly through the user cost of capital and the finance-attenuated elasticity D and wealth-balance-sheet term E (see Section 3.1). The long-run ratio $K_w(t)/K_p(t)$ then dips slightly below one, and the debt ratio $B_w(t)/B_p(t)$ moves closer to (and just under) one as well, indicating that a mineral-price doubling destroys more capital and induces at least as much deleveraging as an oil-price doubling.

Labor reacts more uniformly across elasticities. For all three values of σ, the employment ratio $L_w(t)/L_p(t)$ hovers very close to one but tends to stay slightly above it throughout the transition and in the new steady state. This is consistent with the mineral-price elasticities when $E > 0$: A higher mineral price raises the user cost of capital and reduces net foreign wealth, prompting households to work a bit more while firms substitute labor for more expensive capital. By contrast, the oil shock mainly alters operating costs and has a weaker balance-sheet component, so labor moves less and generally remains below the path under the mineral shock.

Overall, the exercise yields two robust insights. First, for all the elasticities we consider, a mineral-price hike generates smaller welfare and output losses than an oil-price hike of the same magnitude. Second, the channel through which the economy absorbs the mineral shock depends critically on σ. When inputs are hard to substitute, the economy leans more on external borrowing to preserve its capital base, relying more on external borrowing, i.e., deleveraging less, in exchange for smoother real activity. When inputs are readily substitutable, firms cut investment more aggressively after a mineral price shock, thereby lowering both capital and debt and allowing real activity to converge with only modest welfare differentials. Policymakers contemplating a mineral-intensive transition should therefore pay close attention to the flexibility of domestic production. The less adaptable the technology, the more a mineral shock manifests as an external-balance problem; the more adaptable it is, the more the shock plays out through domestic investment dynamics.

3.2.3 Sensitivity to access to world financial markets

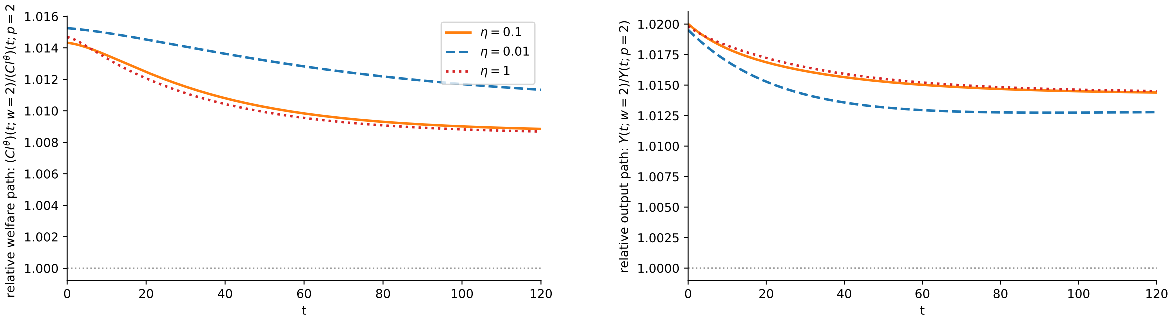
Figure 3 illustrates how access to world financial markets shapes the relative transition paths after a mineral price doubling versus an oil price doubling. Each panel reports the ratio $X_w(t)/X_p(t)$ for $X \in \{W, Y, K, L, C, B\}$, comparing the mineral shock ($w : 1 \to 2$) to the oil shock ($p : 1 \to 2$). Curves correspond to three values of the borrowing-premium parameter $\eta \in \{0.01, 0.10, 1.00\}$, holding all other parameters at their OECD benchmark values ($\sigma = 1.25$). In the plots, the orange solid line refers to $\eta = 0.10$, the blue dashed line to $\eta = 0.01$, and the red dotted line to $\eta = 1.00$. Values above one indicate that the mineral shock is less contractionary for that variable than the oil shock at that horizon.

With tight credit markets (high risk-premium steepness, $\eta = 1.00$), the red dotted lines show that the qualitative ordering between the two shocks is unchanged. Output remains lower under the oil shock than under the mineral shock at all horizons. Welfare, consumption, and employment also behave more favorably under the mineral shock: the ratios $W_w(t)/W_p(t)$ and $C_w(t)/C_p(t)$ lie modestly above one, and the labor ratio $L_w(t)/L_p(t)$ stays very close to, but typically just above, one. By contrast, the capital and debt ratios sit below one throughout the transition. This shows that even when borrowing is costly, the mineral shock still induces a somewhat larger reduction in the capital stock and a slightly stronger deleveraging than the oil shock, whereas the oil shock remains more recessionary for output and welfare.

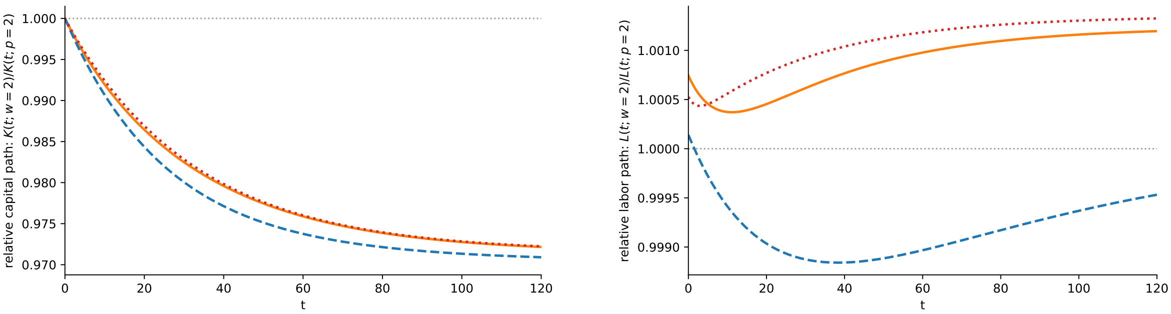
When access to world financial markets is nearly frictionless ($\eta = 0.01$, blue dashed lines), the same ranking between the two shocks is preserved and the three η-curves almost overlap. Welfare, output, and consumption ratios again lie above one, indicating smaller welfare and consumption losses from the mineral shock. Employment differences remain negligible, with $L_w(t)/L_p(t)$ hovering extremely close to unity. The main effect of easier external finance is to soften the overall impact of both shocks rather than to change their relative ordering: Long-run welfare and output losses are slightly smaller in magnitude, whereas the mineral shock continues to be absorbed through a deeper contraction in capital and a somewhat stronger reduction in external debt than under the oil shock (see Table 3).

Figure 3: Sensitivity to access to world financial markets: mineral vs oil price doubles

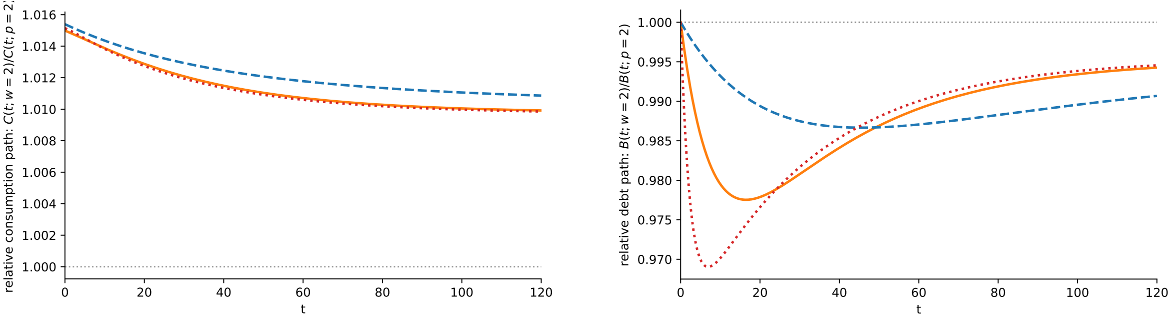
(a) Welfare ratio $W_w(t)/W_p(t)$ (b) Output ratio $Y_w(t)/Y_p(t)$



(c) Capital ratio $K_w(t)/K_p(t)$ (d) Employment ratio $L_w(t)/L_p(t)$



(e) Consumption ratio $C_w(t)/C_p(t)$ (f) Debt ratio $B_w(t)/B_p(t)$



Notes: OECD-type economy calibration with $\sigma = 1.25$. Each panel shows the ratio of the transitional path after a mineral price doubling to the path after an oil price doubling, for three values of the borrowing-premium parameter $\eta \in \{0.01, 0.10, 1.00\}$ (see legend).

Thus, for the OECD-type calibration with $\sigma = 1.25$, varying η over an order of magnitude leaves the central message intact: mineral price spikes are consistently less recessionary for welfare, output, and consumption.

3.2.4 Same-budget-debt comparison: doubling oil price vs. raising mineral price

To isolate the financial channel, we now compare a doubling of the oil price with a mineral-price increase that delivers the *same new steady-state debt* as the oil shock. For each pair (σ, η) we choose a new mineral price $w^* > 0$ such that the long-run level of

Table 3: Short-run and long-run percentage changes under alternative oil and mineral price shocks, by elasticity of substitution σ and borrowing-premium parameter η. First data row reports the new mineral price w^* that yields the same steady-state debt as when p doubles.

		$\sigma = 0.9$			$\sigma = 1$			$\sigma = 1.25$		
Metric	Scenario	$\eta = 0.01$	$\eta = 0.10$	$\eta = 1.00$	$\eta = 0.01$	$\eta = 0.10$	$\eta = 1.00$	$\eta = 0.01$	$\eta = 0.10$	$\eta = 1.00$
w^*	same-budget debt	3.665	3.797	3.807	2.937	3.046	3.055	1.799	1.831	1.834
$\Delta Y(0)\%$	$p : 1 \to 2$	-3.34	-4.29	-4.50	-2.83	-3.42	-3.56	-1.68	-1.86	-1.91
$\Delta Y(0)\%$	$w : 1 \to 2$	0.43	0.24	0.16	0.37	0.20	0.12	0.24	0.10	0.03
$\Delta Y(0)\%$	w^* (same-debt)	1.13	0.68	0.45	0.71	0.40	0.23	0.19	0.08	0.03
$\Delta \tilde{Y}\%$	$p : 1 \to 2$	-8.41	-8.60	-8.61	-5.61	-5.61	-5.61	-2.40	-2.37	-2.36
$\Delta \tilde{Y}\%$	$w : 1 \to 2$	-2.67	-2.59	-2.59	-1.99	-1.84	-1.83	-1.13	-0.97	-0.96
$\Delta \tilde{Y}\%$	w^* (same-debt)	-6.81	-6.94	-6.95	-3.76	-3.67	-3.66	-0.91	-0.81	-0.80
$\Delta W(0)\%$	$p : 1 \to 2$	-9.80	-5.93	-5.30	-5.89	-4.01	-3.64	-2.01	-1.61	-1.51
$\Delta W(0)\%$	$w : 1 \to 2$	-1.66	-0.73	-0.48	-1.12	-0.50	-0.29	-0.52	-0.20	-0.06
$\Delta W(0)\%$	w^* (same-debt)	-4.35	-2.05	-1.34	-2.15	-1.01	-0.58	-0.42	-0.16	-0.05
$\Delta \tilde{W}\%$	$p : 1 \to 2$	-9.29	-9.09	-9.08	-5.62	-5.62	-5.62	-1.93	-1.98	-1.98
$\Delta \tilde{W}\%$	$w : 1 \to 2$	-2.97	-3.05	-3.06	-2.00	-2.18	-2.19	-0.91	-1.13	-1.15
$\Delta \tilde{W}\%$	w^* (same-debt)	-7.55	-8.10	-8.14	-3.77	-4.33	-4.38	-0.73	-0.94	-0.96
$\Delta K(0)\%$	$p : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta K(0)\%$	$w : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta K(0)\%$	w^* (same-debt)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \tilde{K}\%$	$p : 1 \to 2$	-8.41	-8.60	-8.61	-5.61	-5.61	-5.61	-2.40	-2.37	-2.36
$\Delta \tilde{K}\%$	$w : 1 \to 2$	-5.65	-5.57	-5.57	-5.32	-5.17	-5.16	-5.31	-5.15	-5.14
$\Delta \tilde{K}\%$	w^* (same-debt)	-14.02	-14.46	-14.50	-9.89	-10.12	-10.14	-4.28	-4.32	-4.32
$\Delta L(0)\%$	$p : 1 \to 2$	2.44	0.56	0.15	1.25	0.28	0.04	0.28	0.05	-0.01
$\Delta L(0)\%$	$w : 1 \to 2$	0.80	0.45	0.30	0.58	0.32	0.18	0.30	0.13	0.04
$\Delta L(0)\%$	w^* (same-debt)	2.13	1.28	0.85	1.12	0.64	0.37	0.24	0.11	0.03
$\Delta \tilde{L}\%$	$p : 1 \to 2$	0.00	-0.21	-0.23	0.00	0.00	0.00	0.00	0.04	0.04
$\Delta \tilde{L}\%$	$w : 1 \to 2$	0.00	0.08	0.09	0.00	0.16	0.17	0.00	0.16	0.18
$\Delta \tilde{L}\%$	w^* (same-debt)	0.00	0.21	0.23	0.00	0.31	0.34	0.00	0.13	0.15
$\Delta C(0)\%$	$p : 1 \to 2$	-7.56	-5.56	-5.20	-4.71	-3.80	-3.61	-1.73	-1.56	-1.52
$\Delta C(0)\%$	$w : 1 \to 2$	-0.87	-0.41	-0.27	-0.54	-0.26	-0.15	-0.22	-0.08	-0.03
$\Delta C(0)\%$	w^* (same-debt)	-2.28	-1.16	-0.77	-1.05	-0.52	-0.30	-0.18	-0.07	-0.02
$\Delta \tilde{C}\%$	$p : 1 \to 2$	-9.29	-9.22	-9.22	-5.62	-5.62	-5.62	-1.93	-1.94	-1.95
$\Delta \tilde{C}\%$	$w : 1 \to 2$	-2.97	-3.00	-3.00	-1.99	-2.06	-2.07	-0.91	-0.99	-0.99
$\Delta \tilde{C}\%$	w^* (same-debt)	-7.54	-7.97	-8.00	-3.77	-4.10	-4.13	-0.73	-0.82	-0.83

external debt after the w-shock coincides with the long-run debt level after p doubles. Table 3 reports, for all (σ, η) combinations, the implied w^* and the short-run and long-run percentage changes in output, welfare, capital, employment, and consumption under three scenarios: an oil-price doubling ($p : 1 \to 2$), a mineral-price doubling ($w : 1 \to 2$), and the same-budget-debt mineral shock ($w : 1 \to w^*$). Additional outcomes (including the shadow price of capital, the interest rate, and the oil input) are provided in Appendix Table A1.

Two patterns stand out. First, except in the OECD-type case with highly flexible production ($\sigma = 1.25$), the mineral price has to move by *more* than 100% to replicate the debt effect of a doubling of p. When $\sigma \approx 1$ and $\eta = 0.10$, matching the steady-state debt requires $w^* \approx 3.05$, and when $\sigma = 0.9$ the required increase is close to 280–300% ($w^* \approx 3.7$–3.8). Only for $\sigma = 1.25$ does the same-debt mineral price lie below a doubling, at $w^* \approx 1.80$–1.83. This aligns with the message from Section 3.1: mineral-price shocks interact much more weakly with the risk premium than oil-price shocks unless the elasticity of substitution is high.

Second, even under this stricter same-debt metric, the mineral shock remains less damaging for real activity and welfare than the oil shock. In the OECD benchmark ($\sigma = 1.25, \eta = 0.10$), doubling p reduces long-run output by about 2.4% and welfare by about 2.0%, whereas raising w only to 1.83 to hit the same debt target yields a long-run output loss of roughly 0.8% and a welfare loss just below 1%. For less flexible technologies the gap narrows, because w^* becomes very large and the mineral shock must be strong enough to compress capital and raise q substantially, but even at $\sigma = 0.9$ the long-run output and welfare losses from the same-debt mineral shock remain somewhat smaller than those from the oil-price doubling.

Overall, the same-budget-debt exercise reinforces the main conclusion from the raw p-vs.-w comparison: Even when we force the external balance sheet to react in the same way, mineral-price shocks are systematically less recessionary for output and welfare than oil-price shocks.

3.3 Assumptions, interpretation, and policy discussion

Our quantitative results should be read in light of two simplifying assumptions that make our estimates an upper bound on the macroeconomic impact of mineral price shocks. First, we treat the mineral input as a single composite whose price rises uniformly. In reality, critical minerals are geographically concentrated, used in different technologies, and likely to experience staggered demand cycles and heterogeneous supply conditions; see International Energy Agency (2022); Arrobas et al. (2023). Our scenario in which *all* relevant minerals simultaneously undergo a large, permanent price increase is therefore deliberately extreme and is best viewed as bracketing the upper tail of possible outcomes rather than describing the most probable path. Second, the model abstracts from substitution across materials and technologies in capital formation. We assume that there are no viable alternatives to the benchmark mineral bundle, so higher mineral prices translate one-for-one into a higher cost of installing new capital. In practice, firms and innovators respond to sustained price increases by changing product design, re-optimizing supply chains, and developing new technologies—for example, shifting battery chemistries away from cobalt-intensive designs or developing sodium-ion batteries and rare-earth-lean motor technologies that reduce dependence on specific critical inputs; see United Nations Conference on Trade and Development (2025); Yao, Benson and Chueh (2025); Rangarajan et al. (2025). These substitution margins, which are likely to expand as the transition unfolds, would dampen both the long-run responses and the transitional losses we compute.

Within this interpretative frame, the analysis points to a simple but robust ranking:

Across a wide range of technologies and financial environments, an oil-price increase is more damaging for aggregate activity and welfare than a mineral-price increase of the same size. In the long run, the elasticities in Section 3.1 show that output, consumption, and welfare respond more negatively to oil price than to mineral price for all calibrations that keep the economy intertemporally viable. The numerical simulations in Section 3.2 confirm that this ranking carries over to short-run impacts and transitional dynamics.

Thus, fears that exposure to critical minerals will mechanically replicate the macroeconomic vulnerability associated with oil dependence should be qualified. In our framework, mineral-price shocks operate primarily through the user cost of capital and the external balance sheet. They primarily shift adjustment toward investment and the external balance sheet, while generating milder recessions in output and welfare than comparable oil shocks. Whether investment and external debt adjust more or less than under an oil shock depends on production substitutability (Figure 2). This suggests that advanced, capital-rich economies can accommodate sizable movements in mineral prices, provided that their financial systems and policy frameworks can absorb fluctuations in investment and the current account. The public-finance literature on resource-rich economies similarly emphasizes the role of stabilization funds, precautionary savings, and conservative fiscal rules in smoothing commodity-driven revenue and balance-sheet volatility; see Davis et al. (2001); Shabsigh and Ilahi (2007); Sugawara (2014); Taguchi and Ganbayar (2022). In line with this evidence, macroprudential and fiscal tools that smooth balance-sheet adjustments (for example, countercyclical capital buffers, precautionary wealth buffers such as foreign-exchange and fiscal reserves, or stabilization funds) are therefore more natural instruments than policies aimed at offsetting short-run demand losses.

The sensitivity experiments in Sections 3.2.2 and 3.2.3 further show that the relative impact of mineral versus oil shocks depends critically on production flexibility and access to external finance. A higher elasticity of substitution allows firms to reallocate away from expensive inputs, but mineral shocks remain relatively less recessionary for output and welfare. Thus, policies that raise effective substitutability—through diversification of energy inputs, support for flexible production technologies, and reforms that ease factor reallocation—reduce exposure to both oil spikes and mineral shocks, echoing the emphasis in recent transition scenarios on technology diversification, material efficiency, and recycling as key tools for managing critical mineral risks(e.g. International Energy Agency, 2022).

Meanwhile, better access to world financial markets scales down the overall losses without overturning the ranking between the two shocks. In our setting, looser external borrowing constraints also reduce the risk-premium component of the user cost of capital, which matters more for mineral shocks because minerals directly change the cost of capital through the installation cost. This resonates with the broader view that robust, well-regulated access to international capital markets and adequate precautionary buffers help economies absorb commodity price swings through financial portfolios and investment adjustments rather than through abrupt contractions in output and consumption; see Bianchi, Hatchondo and Martinez (2018); Marioli and Vasishtha (2025). Thus, preserving prudent but reliable access to external finance helps ensure that mineral-price increases are absorbed mainly through manageable adjustments in investment and the external balance sheet, rather than through disorderly contractions in output and consumption.

4 Conclusion

This paper has developed a neoclassical open-economy growth model in which oil is a flow input in production, critical minerals are embodied in new capital, and external borrowing carries a risk premium that rises with the country's debt position. We compare, within a unified setting, how permanent oil and mineral price shocks affect output, consumption, welfare, capital, and labor. We derive closed-form steady-state elasticities with respect to both prices and then use numerical simulations to trace out transitional dynamics under alternative assumptions about production flexibility and access to world financial markets.

The analysis shows that oil and mineral price shocks affect the economy through different channels and with different magnitudes. A sustained increase in the oil price systematically reduces output, capital, oil use, and consumption, with the size of these effects largely determined by production technology and the ease of substituting away from oil. By contrast, higher mineral prices act mainly through the cost of installing new capital and through the wealth balance sheet of the economy. They contract the capital stock and alter the debt position, but have smaller and more gradual effects on economic activity and welfare. For all calibrations that keep the economy intertemporally viable, the losses in output and welfare associated with mineral price shocks are consistently smaller than those associated with oil price shocks.

These findings carry clear policy implications. Greater reliance on critical minerals does not replicate the macroeconomic vulnerability associated with oil dependence one-for-one. Mineral-price shocks primarily transmit through investment and the external balance sheet, whereas oil-price shocks remain the more severe threat to aggregate activity and welfare. Policies that increase effective substitutability in production and preserve prudent but reliable access to external finance are therefore central to managing both types of shocks. In our flexible-price framework, macroprudential, fiscal, and reserve-management tools that smooth investment and current-account adjustments, rather than short-run demand management, are natural instruments for managing mineral-price shocks. Extending the analysis to environments with nominal rigidities and to commodity-exporting economies is an important direction for future research, to assess how monetary-policy tradeoffs, exchange-rate pass-through, and terms-of-trade gains modify these policy prescriptions and potentially reshape the relative impacts of oil versus mineral shocks.

At the same time, the model is deliberately stylized. We treat "critical minerals" as a single composite input with an exogenous price and represent production with one aggregate sector and a representative agent. Relaxing these assumptions—by introducing stochastic oil and mineral prices, allowing for substitution across mineral chemistries and technologies, incorporating multiple sectors and heterogeneous agents, and embedding the analysis in a multi-country setting with explicit climate and technology policies—would help connect the mechanisms highlighted here to the more complex realities of a global, low-carbon, mineral-intensive energy transition.

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Online Appendices

A Derivation of the equilibrium dynamics

Combining the optimality conditions with the capital accumulation equation and the flow budget constraint, the dynamics can be expressed as an autonomous system in the four stationary variables (K, B, q, l).

First, using capital accumulation and the pricing condition for new capital, we can write the evolution of the per-capita capital stock as

$$\frac{\dot{K}}{K} = \frac{m^2\big(q - w\big)}{h} - (n + \delta). \tag{A1}$$

Second, the oil demand implied by the intratemporal production decision can be expressed as a function of capital and leisure,

$$Z = \varphi(K, l), \tag{A2}$$

while optimal consumption can be written, using the labor-leisure FOC, as

$$C = \frac{l}{\theta} F_L\big(K, 1 - l, \varphi(K, l)\big) \;\equiv\; \psi(K, l). \tag{A3}$$

Substituting $Z = \varphi(K, l)$, $C = \psi(K, l)$, and investment from the pricing condition into the flow budget constraint yields the law of motion for external debt,

$$\dot{B} = \left[r\left(\tfrac{B}{qK}\right) - n\right]B + \psi(K, l) + p\,\varphi(K, l) + \frac{m^2q^2 - m^2w^2}{2h}K - F\big(K, 1 - l, \varphi(K, l)\big). \tag{A4}$$

Third, the intertemporal arbitrage condition for capital can be written as

$$\frac{\dot{q}}{q} = r\left(\tfrac{B}{qK}\right) + \delta - \frac{m^2(q - w)^2}{2hq} - \frac{F_K\big(K, 1 - l, \varphi(K, l)\big)}{q}. \tag{A5}$$

Finally, differentiating the consumption FOC, using the Euler equation and the definition of $\psi(K, l)$, gives the adjustment of leisure,

$$\frac{\dot{l}}{l} = \frac{\left[r\left(\tfrac{B}{qK}\right) - \beta - n\right] \;-\; (1 - \gamma)\frac{\psi_K(K,l)K}{\psi(K,l)}\left[\frac{m^2(q-w)}{h} - (n + \delta)\right]}{(1 - \gamma)\frac{\psi_l(K,l)l}{\psi(K,l)} - \gamma\theta}. \tag{A6}$$

Equations (A1)-(A6) fully characterise the macroeconomic transition. The system is nonlinear and does not admit a closed-form solution; in Section 3.2 we study its behaviour numerically for the CES specification of $F(\cdot)$.

Table A1: Short-run and long-run percentage changes under alternative oil and mineral price shocks, by elasticity of substitution σ and borrowing-premium parameter η. First data row reports the new mineral price w^* that yields the same steady-state debt as when p doubles.

		$\sigma = 0.9$			$\sigma = 1$			$\sigma = 1.25$		
Metric	Scenario	$\eta = 0.01$	$\eta = 0.10$	$\eta = 1.00$	$\eta = 0.01$	$\eta = 0.10$	$\eta = 1.00$	$\eta = 0.01$	$\eta = 0.10$	$\eta = 1.00$
w^*	same-budget debt	3.665	3.797	3.807	2.937	3.046	3.055	1.799	1.831	1.834
$\Delta Y(0)\%$	$p : 1 \to 2$	-3.34	-4.29	-4.50	-2.83	-3.42	-3.56	-1.68	-1.86	-1.91
$\Delta Y(0)\%$	$w : 1 \to 2$	0.43	0.24	0.16	0.37	0.20	0.12	0.24	0.10	0.03
$\Delta Y(0)\%$	w^* (same-debt)	1.13	0.68	0.45	0.71	0.40	0.23	0.19	0.08	0.03
$\Delta \tilde{Y}\%$	$p : 1 \to 2$	-8.41	-8.60	-8.61	-5.61	-5.61	-5.61	-2.40	-2.37	-2.36
$\Delta \tilde{Y}\%$	$w : 1 \to 2$	-2.67	-2.59	-2.59	-1.99	-1.84	-1.83	-1.13	-0.97	-0.96
$\Delta \tilde{Y}\%$	w^* (same-debt)	-6.81	-6.94	-6.95	-3.76	-3.67	-3.66	-0.91	-0.81	-0.80
$\Delta W(0)\%$	$p : 1 \to 2$	-9.80	-5.93	-5.30	-5.89	-4.01	-3.64	-2.01	-1.61	-1.51
$\Delta W(0)\%$	$w : 1 \to 2$	-1.66	-0.73	-0.48	-1.12	-0.50	-0.29	-0.52	-0.20	-0.06
$\Delta W(0)\%$	w^* (same-debt)	-4.35	-2.05	-1.34	-2.15	-1.01	-0.58	-0.42	-0.16	-0.05
$\Delta \tilde{W}\%$	$p : 1 \to 2$	-9.29	-9.09	-9.08	-5.62	-5.62	-5.62	-1.93	-1.98	-1.98
$\Delta \tilde{W}\%$	$w : 1 \to 2$	-2.97	-3.05	-3.06	-2.00	-2.18	-2.19	-0.91	-1.13	-1.15
$\Delta \tilde{W}\%$	w^* (same-debt)	-7.55	-8.10	-8.14	-3.77	-4.33	-4.38	-0.73	-0.94	-0.96
$\Delta K(0)\%$	$p : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta K(0)\%$	$w : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta K(0)\%$	w^* (same-debt)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \tilde{K}\%$	$p : 1 \to 2$	-8.41	-8.60	-8.61	-5.61	-5.61	-5.61	-2.40	-2.37	-2.36
$\Delta \tilde{K}\%$	$w : 1 \to 2$	-5.65	-5.57	-5.57	-5.32	-5.17	-5.16	-5.31	-5.15	-5.14
$\Delta \tilde{K}\%$	w^* (same-debt)	-14.02	-14.46	-14.50	-9.89	-10.12	-10.14	-4.28	-4.32	-4.32
$\Delta L(0)\%$	$p : 1 \to 2$	2.44	0.56	0.15	1.25	0.28	0.04	0.28	0.05	-0.01
$\Delta L(0)\%$	$w : 1 \to 2$	0.80	0.45	0.30	0.58	0.32	0.18	0.30	0.13	0.04
$\Delta L(0)\%$	w^* (same-debt)	2.13	1.28	0.85	1.12	0.64	0.37	0.24	0.11	0.03
$\Delta \tilde{L}\%$	$p : 1 \to 2$	0.00	-0.21	-0.23	0.00	0.00	0.00	0.00	0.04	0.04
$\Delta \tilde{L}\%$	$w : 1 \to 2$	0.00	0.08	0.09	0.00	0.16	0.17	0.00	0.16	0.18
$\Delta \tilde{L}\%$	w^* (same-debt)	0.00	0.21	0.23	0.00	0.31	0.34	0.00	0.13	0.15
$\Delta C(0)\%$	$p : 1 \to 2$	-7.56	-5.56	-5.20	-4.71	-3.80	-3.61	-1.73	-1.56	-1.52
$\Delta C(0)\%$	$w : 1 \to 2$	-0.87	-0.41	-0.27	-0.54	-0.26	-0.15	-0.22	-0.08	-0.03
$\Delta C(0)\%$	w^* (same-debt)	-2.28	-1.16	-0.77	-1.05	-0.52	-0.30	-0.18	-0.07	-0.02
$\Delta \tilde{C}\%$	$p : 1 \to 2$	-9.29	-9.22	-9.22	-5.62	-5.62	-5.62	-1.93	-1.94	-1.95
$\Delta \tilde{C}\%$	$w : 1 \to 2$	-2.97	-3.00	-3.00	-1.99	-2.06	-2.07	-0.91	-0.99	-0.99
$\Delta \tilde{C}\%$	w^* (same-debt)	-7.54	-7.97	-8.00	-3.77	-4.10	-4.13	-0.73	-0.82	-0.83
$\Delta B(0)\%$	$p : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta B(0)\%$	$w : 1 \to 2$	-0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	0.00
$\Delta B(0)\%$	w^* (same-debt)	0.00	0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \tilde{B}\%$	$p : 1 \to 2$	-8.41	-8.60	-8.61	-5.61	-5.61	-5.61	-2.40	-2.37	-2.36
$\Delta \tilde{B}\%$	$w : 1 \to 2$	-3.34	-3.26	-3.25	-3.00	-2.84	-2.83	-2.98	-2.83	-2.81
$\Delta \tilde{B}\%$	w^* (same-debt)	-8.41	-8.60	-8.61	-5.61	-5.61	-5.61	-2.40	-2.37	-2.36
$\Delta q(0)\%$	$p : 1 \to 2$	-4.60	-4.46	-3.51	-3.27	-3.09	-2.55	-1.45	-1.34	-1.19
$\Delta q(0)\%$	$w : 1 \to 2$	-0.76	-0.26	0.27	-0.74	-0.30	0.13	-0.82	-0.49	-0.22
$\Delta q(0)\%$	w^* (same-debt)	-2.51	-1.16	0.44	-1.62	-0.79	0.13	-0.64	-0.39	-0.17
$\Delta \tilde{q}\%$	$p : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \tilde{q}\%$	$w : 1 \to 2$	2.45	2.45	2.45	2.45	2.45	2.45	2.45	2.45	2.45
$\Delta \tilde{q}\%$	w^* (same-debt)	6.53	6.86	6.88	4.75	5.01	5.04	1.96	2.04	2.05
$\Delta r(0)\%$	$p : 1 \to 2$	0.88	0.85	0.67	0.62	0.58	0.48	0.27	0.25	0.22
$\Delta r(0)\%$	$w : 1 \to 2$	0.14	0.05	-0.05	0.14	0.05	-0.02	0.15	0.09	0.04
$\Delta r(0)\%$	w^* (same-debt)	0.47	0.21	-0.08	0.30	0.15	-0.02	0.12	0.07	0.03
$\Delta \tilde{r}\%$	$p : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \tilde{r}\%$	$w : 1 \to 2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta \tilde{r}\%$	w^* (same-debt)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta Z(0)\%$	$p : 1 \to 2$	-48.20	-48.71	-48.82	-51.41	-51.70	-51.78	-58.66	-58.74	-58.76
$\Delta Z(0)\%$	$w : 1 \to 2$	0.43	0.24	0.16	0.37	0.20	0.12	0.24	0.10	0.03
$\Delta Z(0)\%$	w^* (same-debt)	1.13	0.68	0.45	0.71	0.40	0.23	0.19	0.08	0.03
$\Delta \tilde{Z}\%$	$p : 1 \to 2$	-50.92	-51.02	-51.03	-52.80	-52.80	-52.80	-58.97	-58.95	-58.95
$\Delta \tilde{Z}\%$	$w : 1 \to 2$	-2.67	-2.59	-2.59	-1.99	-1.84	-1.83	-1.13	-0.97	-0.96
$\Delta \tilde{Z}\%$	w^* (same-debt)	-6.81	-6.94	-6.95	-3.76	-3.67	-3.66	-0.91	-0.81	-0.80

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