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Primer**

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Robert S. Pindyck

01-002 WP

May 2001

**A Joint Center of the Department of Economics, Laboratory for Energy
and the Environment, and Sloan School of Management**

THE DYNAMICS OF COMMMODITY SPOT AND FUTURES MARKETS: A PRIMER^{*}

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Robert S. Pindyck

Massachusetts Institute of Technology
Cambridge, MA 02142

This draft: May 16, 2001

Abstract: I discuss the short-run dynamics of commodity prices, production, and inventories, as well as the sources and effects of market volatility. I explain how prices, rates of production, and inventory levels are interrelated, and are determined via equilibrium in two interconnected markets: a cash market for spot purchases and sales of the commodity, and a market for storage. I show how equilibrium in these markets affects and is affected by changes in the level of price volatility. I also explain the role and behavior of commodity futures markets, and the relationship between spot prices, futures prices, and inventory behavior. I illustrate these ideas with data for the petroleum complex – crude oil, heating oil, and gasoline – over the past two decades.

JEL Classification Numbers: G13; L71, Q40

Keywords: Commodity prices, futures markets, inventories, volatility.

^{*} This paper was written while the author was a Visiting Professor at the Harvard Business School, and the hospitality of that institution is gratefully acknowledged. I am also grateful to M.I.T.'s Center for Energy and Environmental Policy Research for its financial support of the research leading to this paper, to Denny Ellerman, Geoffrey Pearce, and G. Campbell Watkins for helpful comments and suggestions, and to Nicola Stafford for her outstanding research assistance.

1. Introduction

The markets for oil products, natural gas, and many commodities are characterized by high levels of volatility. Prices and inventory levels fluctuate considerably from week to week, in part predictably (e.g., due to seasonal shifts in demand) and in part unpredictably. Furthermore, levels of volatility themselves vary over time. This paper discusses the short-run dynamics of commodity prices, production, and inventories, as well as the sources and effects of market volatility. I explain how prices, rates of production, and inventory levels are interrelated, and are determined via equilibrium in two interconnected markets: a cash market for spot purchases and sales of the commodity, and a market for storage. I also explain how equilibrium in these markets affects and is affected by changes in the level of price volatility.

Because commodity markets are volatile, producers and consumers often seek ways of hedging and trading risk. In response to this need, markets for commodity risk trading arose, and their use has become increasingly widespread. Instruments traded in these markets include futures and forward contracts, options, swaps, and other derivatives. Futures contracts are among the most important of these instruments, and provide important information about cash and storage markets. Thus another goal of this paper is to explain the role and operation of commodity futures markets, and the relationship between spot prices, futures prices, and inventory behavior.

Understanding the behavior and role of volatility is important in its own right. Price volatility drives the demand for hedging, whether it is done via financial instruments such as futures contracts or options, or via physical instruments such as inventories. Volatility is a key determinant of the values of commodity-based contingent claims, such as futures contracts, options on futures, and commodity production facilities, such as oil wells, refineries, and pipelines. (Indeed, such production facilities can usefully be viewed as call options on the commodity itself.) Furthermore, as I will explain, volatility plays an important role in driving short-run commodity cash and storage market dynamics.

In markets for storable commodities such as oil, inventories play a crucial role in price formation. As in manufacturing industries, inventories are used to reduce costs of changing production in response to fluctuations (predictable or otherwise) in demand, and

to reduce marketing costs by helping to ensure timely deliveries and avoid stockouts.¹ Producers must determine their production levels jointly with their expected inventory drawdowns or buildups. These decisions are made in light of two prices – a spot price for sale of the commodity itself, and a price for storage. As I will explain, although the price of storage is not directly observed, it can be determined from the spread between futures and spot prices. This price of storage is equal to the marginal value of storage, i.e., the flow of benefits to inventory holders from a marginal unit of inventory, and is termed the *marginal convenience yield*.

Thus there are two interrelated markets for a commodity: the *cash market* for immediate, or “spot,” purchase and sale, and the *storage market* for inventories held by both producers and consumers of the commodity.

In the next section, I begin by discussing the determinants of supply and demand in each of these two markets. I also discuss the nature and characteristics of marginal convenience yield, and I explain how the cash and storage markets are interconnected. In Section 3, I discuss the nature of short-run dynamic adjustment in these two markets, and how these markets respond to shocks to consumption, production, and price volatility. Section 4 introduces futures markets, and explains how marginal convenience yield can be measured from the spread between futures and spot prices. In Section 5, I discuss the behavior and determinants of price volatility, focusing in particular on the petroleum complex. I explain how changes in volatility affect futures prices, production, the demand for storage, and the values of commodity-based options, including such “real options” as production facilities. Section 6 concludes.

Although I discuss some of the basic concepts, I should stress that this paper is not intended to serve as a “short textbook” on commodity futures markets.² My emphasis is primarily on the dynamics of real market variables: production, inventories, and spot prices. However, futures markets are central to this discussion, for two reasons. First, futures contracts and inventories can both be used as a vehicle for reducing risk or the

¹ There have been many studies of inventory behavior in manufacturing industries. Examples of such studies that focus on cost-reducing benefits of inventories include Kahn (1987, 1992), Miron and Zeldes (1988), and Ramey (1991). In manufacturing industry studies, however, one does not have the benefit of futures market data from which one can directly measure the marginal value, i.e., price, of storage.

² There are already several good textbooks on this subject. See, for example, Duffie (1989) and Hull (1997). For other perspectives on futures markets, see Carlton (1984) and Williams (1986).

impact of risk. Second, the spread between the futures price and the spot price gives a direct measure of the marginal value of storage for a commodity. Thus we will need the information from futures prices to measure the value of storage.

To illustrate the ideas developed in this paper, I draw on weekly data for the three interrelated commodities that comprise the petroleum complex: crude oil, heating oil, and gasoline. In a related paper (Pindyck (2001)), I used this data to estimate structural models describing the joint dynamics of inventories, spot and futures prices, and volatility for each of these commodities. Here, my aim is to explain the economic relationships underlying such models, and show how those relationships are manifested in the underlying data.

2. Cash Markets and Storage Markets

In a competitive commodity market subject to stochastic fluctuations in production and/or consumption, producers, consumers, and possibly third parties will hold inventories. These inventories serve a number of functions. Producers hold them to reduce costs of adjusting production over time, and also to reduce marketing costs by facilitating production and delivery scheduling and avoiding stockouts. If marginal production costs are increasing with the rate of output and if demand is fluctuating, producers can reduce their costs over time by selling out of inventory during high-demand periods, and replenishing inventories during low-demand periods. Even if marginal production cost is constant with respect to output, there may be adjustment costs, i.e., costs of changing the rate of production. For example, there may be costs of hiring and training new or temporary workers, leasing additional capital, etc. Selling out of inventory during high-demand periods can reduce these adjustment costs. In addition, inventories are needed as a “lubricant” to facilitate scheduling and reduce marketing costs. Industrial consumers also hold inventories, and for the same reasons – to reduce adjustment costs and facilitate production (i.e., when the commodity is used as a production input), and to avoid stockouts.

To the extent that inventories can be used to reduce production and marketing costs in the face of fluctuating demand conditions, they will have the effect of reducing the magnitude of short-run market price fluctuations. Also, because it is costly for firms to

reduce inventory holdings beyond some minimal level, and because inventories can never become negative, we would expect that price volatility would be greater during periods when inventories are low. This is indeed what we observe in the data.

When inventory holdings can change, production in any period need not equal consumption. As a result, the market-clearing price is determined not only by current production and consumption, but also by changes in inventory holdings. Thus, to understand commodity market behavior, we must account for equilibrium in both the *cash* and *storage* markets.

2.1. The Cash Market

In the cash market, purchases and sales of the commodity for immediate delivery occur at a price that we will refer to as the “spot price.” (As I explain later, as it is commonly used, the term “cash price” has a somewhat different meaning.) Because inventory holdings can change, the spot price does not equate production and consumption. Instead, we can characterize the cash market as a relationship between the spot price and “net demand,” i.e., the difference between production and consumption.

To see this, note that total demand in the cash market is a function of the spot price, and may also be a function of other variables such as the weather, aggregate income, certain capital stocks (e.g., stocks of automobiles in the case of gasoline and industrial boilers in the case of residual fuel oil or natural gas), and random shocks reflecting unpredictable changes in tastes and technologies. Because of these shocks and the fact that some of the variables affecting demand (such as the weather) are themselves partly unpredictable, demand will fluctuate unpredictably. We can therefore write the demand function for the cash market as:

$$Q = Q(P; z_1, \varepsilon_1) \quad (1)$$

where P is the spot price, z_1 is a vector of demand-shifting variables, and ε_1 is a random shock.

The supply of the commodity in the cash market is likewise a function of the spot price, and also of a set of (partly unpredictable) variables affecting the cost of production, such as energy and other raw material prices, wage rates, and various capital stocks (such as oil rigs, pipelines, and refineries), as well as random shocks reflecting unpredictable

changes in operating efficiency, strikes, etc. Thus the supply function also shifts unpredictably, and will be written as:

$$X = X(P; z_2, \varepsilon_2) \quad (2)$$

where z_2 is a vector of supply-shifting variables, and ε_2 is a random shock.

Letting N_t denote the inventory level, the change in inventories at time t is given by the accounting identity:

$$\Delta N_t = X(P_t; z_{2t}, \varepsilon_{2t}) - Q(P_t; z_{1t}, \varepsilon_{1t}) \quad (3)$$

We can think of ΔN_t as *net demand*, i.e., the demand for production in excess of consumption. Thus this equation just says that the cash market is in equilibrium when net demand equals net supply. We can rewrite this in terms of the following inverse net demand function:

$$P_t = f(\Delta N_t; z_{1t}, z_{2t}, \varepsilon_t) \quad (4)$$

Market clearing in the cash market therefore implies a relationship between the spot price and the change in inventories.

Because $\partial X / \partial P > 0$ and $\partial Q / \partial P < 0$, the inverse net demand function is *upward sloping* in ΔN , i.e., a higher price corresponds to a larger X and smaller Q , and thus a larger ΔN . This is illustrated in Figure 1, where $f_1(\Delta N)$ is the inverse net demand function for some initial set of values for z_1 and z_2 . Note that if $\Delta N = 0$, the spot price is P_1 . In general, of course, ΔN need not be zero.

Now suppose that the commodity of interest is heating oil, and that a spell of unexpected cold weather arrives (shifting z_1). This will cause the inverse net demand function to shift upward; in the figure, the new function is $f_2(\Delta N)$. Suppose the spot price is initially P_1 . What will happen to the price following this shift in net demand? The answer depends on what happens to inventories. If the change in weather is expected to persist for a long time and production responds rapidly to price changes, inventory holdings might remain unchanged, so that the price increases to P_3 . If, as is more likely, the weather is expected to return to normal, inventories will be drawn down (so that $\Delta N < 0$), and the price will only increase to P_2 , as shown in the figure. Eventually, if the weather indeed returns to normal, the price will fall back to P_1 as the net demand function shifts back to $f_1(\Delta N)$.

2.2. The Market for Storage

Now consider the market for storage. At any instant of time, the *supply of storage* is simply the total quantity of inventories held by producers, consumers, or third parties, i.e., N_t . In equilibrium, this quantity must equal the quantity demanded, which – as with any good – is a function of the price. The *price of storage* is the “payment” by inventory holders for the privilege of holding a unit of inventory. (I will explain momentarily how this “payment” occurs.) As with any good or service sold in a competitive market, if the price lies on the demand curve, it is equal to the marginal value of the good or service, i.e., the utility from consuming a marginal unit. In the case of commodity storage, this marginal value is the value of the flow of services accruing from holding the marginal unit of inventory; it is what we usually refer to as *marginal convenience yield*. We will denote the price of storage (marginal convenience yield) by ψ_t , so that the *demand for storage* function can be written as $N(\psi)$.

How does the holder of inventory “pay” this price ψ ? First, there may be a physical cost of storage (e.g., tanks to hold heating oil). Second, even if no change in the spot price is expected, there is an opportunity cost of capital equal to the interest forgone by investing in a unit of commodity at a cost P_t . (These first two components of cost, i.e., the physical storage cost and the forgone interest, are together referred to as the *cost of carry* for a commodity.) Third, the spot price might be expected to fall over the period that the inventory is held (the price of heating oil, for example, typically falls between November and March), and this expected depreciation is an additional component of the opportunity cost of capital. Expected depreciation (and thus the price of storage) will be particularly high following periods of supply disruptions or unusual (but temporary) increases in demand. As we will see in Section 4, we can measure expected depreciation, and thus the price of storage, from futures prices.

The marginal value of storage is likely to be small when the total stock of inventories is large (because one more unit of inventory will be of little extra benefit), but it can rise sharply when the stock becomes very small. Thus we would expect the demand for storage function to be downward sloping and convex, i.e., $N'(\psi) < 0$ and $N''(\psi) > 0$.

The demand for storage depends on the price ψ , but we would expect it to depend on other variables as well. For example, it should depend on current and expected future rates of consumption (or production) of the commodity. Thus if a seasonal increase in demand is expected (as with heating oil in the fall and winter), the demand for storage will increase because producers will need greater inventories to avoid sharp increases in production cost and to make timely deliveries. The demand for storage will also depend on the spot price of the commodity; other things equal, one should be willing to pay more to store a higher-priced good than a lower-priced one. Finally, the demand for storage will depend on the volatility of price, which we denote by σ , and which proxies for market volatility in general. This last variable is particularly important; the demand for storage should be greater the greater is volatility because greater volatility makes scheduling and stockout avoidance more costly. Thus, including a random shock, we can write the demand function as $N(\psi; \sigma, z_3, \varepsilon_3)$, where z_3 includes the spot price, consumption, and any other variables (except for σ) that affect demand.

Finally, rewriting this as an inverse demand function, we can describe equilibrium in the market for storage by the following equation:

$$\psi_t = g(N; \sigma, z_3, \varepsilon_3) \quad (5)$$

Thus market clearing in the storage market implies a relationship between marginal convenience yield (the price of storage) and the demand for storage. This is illustrated Figure 2, which shows the downward-sloping demand function $\psi_1^D(N)$, the supply of inventories N_1 , and the market-clearing price ψ_1 .

Suppose that there is an increase in the level of price volatility, σ . A greater level of volatility should increase the need for inventories to buffer fluctuations in demand and supply in the cash market, whatever the price of storage. Thus the demand for storage curve will shift upwards (from $\psi_1^D(N)$ to $\psi_2^D(N)$ in the figure). What will then happen to the price of storage, ψ ? That depends on what happens to inventories, which in turn will depend on expectations regarding future market conditions. If for some reason (such as an expected seasonal decline in price) total inventories remain fixed, as in the figure, the price will increase to ψ_2 . However, if the total level of inventories increases, as would usually happen, the price of storage would rise by less.

To say more about how the cash price, the price of storage, and inventories will change following an exogenous shock, the cash and storage markets must be put together so that their interrelationship can be examined. I turn to that in the next section.

3. Inventories and Price Dynamics

Equations (4) and (5) describe equilibrium in both the cash and storage markets. This equilibrium is dynamic: Given the value of N_{t-1} and the values for z_1 , z_2 , z_3 , and σ at time t , these equations can be solved for the values of ψ_t , N_t , and P_t . (Note from Figure 2 that equating supply and demand in the storage market determines ψ_t and N_t . Since N_{t-1} is given, we know ΔN_t , and from Figure 1, we can determine P_t .) Then, given this value of N_t and the values for z_1 , z_2 , z_3 , and σ at time $t+1$, we can solve for ψ_{t+1} , N_{t+1} , and P_{t+1} , and so on. If there are no shocks (i.e., no changes in the exogenous variables z_1 , z_2 , z_3 , and σ), the system will reach a steady-state equilibrium in which $\Delta N_t = 0$.

In order to examine this equilibrium in more detail, we will consider how these market variables change in response to a temporary or permanent exogenous shock. We will first consider a temporary shift in demand in the cash market (for example, brought about by a spell of unusually cold weather). Then we will consider the effects of a long-lasting increase in price volatility, σ .

3.1. Temporary Demand Shock

I will first consider the effect of a spell of unusually cold weather on the market for heating oil. Let us assume that market participants expect this spell of cold weather to be temporary. Suppose the spot price of heating oil is currently P_0 , inventories are N_0 , and the convenience yield (price of storage) is ψ_0 . We will assume that until the arrival of the cold weather, the market is in steady-state equilibrium so that there has been no build-up or drawdown of inventories (i.e., $\Delta N = 0$). (See Figure 3.) What should we expect to happen in the cash and storage markets after the cold weather arrives?

The cold weather will increase the consumption demand for heating oil, and thereby cause the net demand function, $f(\Delta N)$, to shift upwards. (In Figure 3, this is shown as a shift from $f_1(\Delta N)$ to $f_2(\Delta N)$.) This will immediately push up the spot price. However,

because the cold weather is viewed as temporary, there will be a drawdown of inventories ($\Delta N_1 < 0$), and this will limit the size of the price increase (from P_0 to P_1 in the figure). While the weather remains cold, inventories will fall (from N_0 to N_1) and the convenience yield will rise (from ψ_0 to ψ_1).

Once warm weather returns, the net demand curve will shift back down. The spot price will fall, but at first not to its original level. The reason is that inventories will be re-accumulated, so that production will have to exceed consumption (i.e., $\Delta N > 0$). Thus in Figure 3, the spot price falls from P_1 to $P_2 > P_0$. This accumulation of inventories will continue until the inventory level returns to N_0 and the convenience yield falls back to ψ_0 . At that point, there will be no further accumulation of inventories ($\Delta N = 0$ again), and the spot price will have fallen back to P_0 .

These changes in the spot price, inventory level, and convenience yield will be accompanied by changes in futures prices and in the futures-spot spread, but we will defer our discussion of the futures market to Section 4. Before moving on, however, it will be useful to examine an empirical example of the events described above.

Such an example occurred in January 1990, when much of the United States experienced unusually cold weather. As a result, there was a sharp but temporary increase in spot heating oil prices, as can be seen from Figure 4, which plots the prices of heating oil and crude oil over the period mid-1988 to mid-1992. Note that the prices of both crude oil and heating oil rose sharply during the second half of 1990 and the beginning of 1991; this was in large part the result of Iraqi invasion of Kuwait and the Persian Gulf War. Thus the increase in the price of heating oil during this period was the result of the sustained increase in the price of crude oil, which directly increased the cost of producing heating oil. During January 1990, however, there was no significant increase in the price of crude oil; only heating oil increased in price.

Figure 5 shows the marginal convenience yield for heating oil, as well as average U.S. heating degree-days, for the same four-year period. (I discuss the computation of marginal convenience yield in Section 4.) In each year the heating degree-days series peaks in January or February, but in 1990 the peak (in January) was higher than in any other year in my 1984 – 2000 sample. Thus January 1990 was indeed an unusually cold month, and this would have temporarily pushed up the demand curve in the cash market.

Observe that there was also a sharp increase in the convenience yield during January 1990. This increase was short-lived, and reflected a movement up and then back down the demand for storage curve, as shown in Figure 3.

3.2. Sustained Increase in Volatility

Now let us consider what would happen if the volatility of spot price fluctuations were to increase, and the change was expected to last for a significant period of time. First, recall that I am using spot price volatility as a proxy for general volatility in the cash market, and indeed, price volatility tends to be correlated with volatility in consumption and production. One of the main causes of price volatility is fluctuations in the net demand function, which in turn results from fluctuations in consumption demand and/or production. Furthermore, price fluctuations themselves (whether caused by fluctuations in net demand or something else, such as speculative buying and selling) will cause consumption and production to fluctuate. Thus an increase in price volatility will be accompanied by an increase in the volatility of production and consumption. This in turn will imply an increase in the demand for storage; at any given price of storage, market participants will want to hold greater inventories in order to buffer these fluctuations in production and consumption. The result will be an upward shift in the demand for storage curve.

This increase in volatility will also result in an upward shift in the net demand curve. The reason is that increased volatility increases the value of producers' *operating options*, i.e., options to produce now (at an "exercise price" equal to marginal production cost and with a "payoff" equal to the spot price), rather than waiting for possible increases or decreases in price. These options add an opportunity cost to current production; namely, the cost of exercising the options rather than preserving them. Thus an increase in volatility increases the opportunity cost of current production, which shifts the net demand curve up.

This is illustrated in Figure 6. In the cash market, the net demand curve shifts upward, from $f_1(\Delta N)$ to $f_2(\Delta N)$. In the storage market, the demand for storage curve shifts upward as the increase in volatility drives up the demand for storage. In the figure, this shift is from $\psi_1(N)$ to $\psi_2(N)$. Given that the supply of storage is initially fixed (at N_0 in

the figure), the price of storage, i.e., marginal convenience yield, will increase (from ψ_0 to ψ_1). In addition, the spot price will rise sharply (from P_0 to P_1 in the figure) because of the shift in the net demand curve and because of movement along the curve as inventories start to be built up. As the inventory level increases (from N_0 to N_1), the rate of change of inventories will drop (from ΔN_1) back to zero, and as this happens the spot price will fall part of the way back (to P_2). In addition, marginal convenience yield will fall part of the way back (to ψ_2). Assuming that this increase in volatility is expected to persist indefinitely, we will have a new equilibrium in which the spot price, the convenience yield, and the level of inventories are all higher than they were at the outset.

Once again, an empirical example may prove helpful. It is difficult to find changes that occurred in a real-world commodity market that were expected to persist “indefinitely.” However, as Figure 7 shows, the volatility of crude oil prices increased sharply during July 1990, and remained high through January 1991, a period of about 6 months. (Prior to July 1990 and after January 1991, the average standard deviation of monthly percentage price changes was about 11 percent; during the period July 1990 – January 1991, this number was 25 percent.) This was accompanied by a sharp increase in the spot price itself, and was, of course, largely the result of the Iraqi invasion of Kuwait.

Observe from Figure 7 that the convenience yield for crude oil also increased sharply in July 1990, and remained high throughout the six-month period. The increases in the spot price and the convenience yield reflect the upward shift in the demand for storage curve in Figure 6, the shift in the net demand curve, and the movement along the net demand curve as inventory accumulation occurs. Inventories gradually increase and inventory accumulation drops, and this is accompanied by drops in both the spot price and convenience yield, although not to their original levels. The original equilibrium was restored only after the Gulf War ended; volatility fell to its original level, and both the net demand curve and demand for storage curve shifted back down.

4. Futures Prices

For commodities with actively traded futures contracts, we can use futures prices to measure the marginal convenience yield. Before explaining how this is done, we must be clear about what a futures price measures, and how it differs from a forward price.

4.1. Forward Prices and Futures Prices

A *forward contract* is an agreement to deliver a specified quantity of a commodity at a specified future date, at a price (the *forward price*) to be paid at the time of delivery. The commodity specifications and point of delivery (as well as the quantity, price, and date of delivery) are spelled out in the contract. There are two parties to a forward contract: the buyer (or long position), who will receive the commodity and pay the forward price, and the seller (or short position), who will deliver the commodity. Forward contracts are often traded directly among producers and industrial consumers of the commodity; in some cases they are traded on organized exchanges (such as the London Metals Exchange).

A *futures contract* is also an agreement to deliver a specified quantity of a commodity at a specified future date, at a price (the *futures price*) to be paid at the time of delivery. Futures contracts are usually traded on organized exchanges, such as the New York Mercantile Exchange, and as a result, tend to be more liquid than forward contracts. Other than this, *a futures contract differs from a forward contract only in that the futures contract is “marked to market,”* which means that there is a settlement and corresponding transfer of funds at the end of each trading day.

Consider, for example, a futures contract for 1000 barrels of crude oil, for delivery six months from now. If the six-month futures price has increased by, say, 40 cents per barrel during trading on Monday, the holder of the long position will receive \$400 from the holder of the short position. If on Tuesday the futures price falls by 20 cents, \$200 will flow in the opposite direction. This daily “settling up” reduces the risk that one of the parties will default on the contract. Payments are based on each day’s *settlement price*, which is the price deemed by the futures exchange to be the market-clearing price at the end of the trading day. On an actively traded contract, the settlement price will be the price of the last trade at the close, or very near the close, of trading. For a less actively traded contract, there may not be any trades at or close to the close of trading; in this case the exchange uses closing prices on active contracts to estimate what the market-clearing price would have been had there been trades.

Because the futures contract is marked to market, the futures price will be greater (less) than the forward price if the risk-free interest rate is stochastic *and* is positively

(negatively) correlated with the spot price.³ For most commodities, however, the difference between the forward and futures prices will be very small. In the case of one-month contracts on heating oil, for example, I have estimated that this difference is less than .01 percent.⁴ Thus in what follows, I will ignore the difference between a forward price and a futures price and treat the two as equivalent. This is convenient for purposes of empirical research because for most commodities, futures contracts are much more actively traded than forward contracts, and futures price data are more readily available.⁵

Although futures and forward contracts specify prices to be paid at the time of delivery, it is not necessary to actually take delivery. In fact, the vast majority of futures contracts are “closed out” or “rolled over” before the delivery date, so the commodity does not change hands. The reason is that these contracts are usually held for hedging or speculation purposes, so that delivery of the commodity is not needed. For example, suppose that in January, an industrial consumer of crude oil is worried about the risk of oil price increases during the coming year. That consumer might take a long futures position in crude oil by buying, say, an appropriate number of July futures contracts, but continue to buy oil on an ongoing basis from his usual source. If the price of oil rises between January and July, the consumer will pay more for his oil, but will enjoy an offsetting gain from the futures position. Likewise, if the price goes down, the consumer will pay less for oil but have an offsetting loss from the futures position. As July approaches, the consumer might roll over his position by selling the July contracts and buying, say, December contracts. As December approaches, the consumer might roll over the position again, or simply close it out by selling the contracts. Throughout, the consumer buys oil normally and never takes delivery on the futures contracts.

³ To see why this is so, note that if the interest rate is non-stochastic, the present value of the expected daily cash flows over the life of the futures contract will equal the present value of the expected payment at termination of the forward contract, so the futures and forward prices must be equal. If the interest rate is stochastic and positively correlated with the price of the commodity (which we would expect to be the case for most industrial commodities), daily payments from price increases will on average be more heavily discounted than payments from price decreases, so the initial futures price must exceed the forward price. For a more detailed discussion of this point, see Cox, Ingersoll, and Ross (1981).

⁴ See Pindyck (1994). French (1983) has compared the futures prices for silver and copper on the Comex with their forward prices on the London Metals Exchange and found that these differences are also very small (about 0.1 percent for three-month contracts).

⁵ This is not the case for electricity, however, where for past few years the forward markets have been much more active than the futures markets.

4.2. Convenience Yield

We can now turn to the calculation of convenience yield from futures and spot prices. Let $\psi_{t,T}$ denote the (capitalized) flow of marginal convenience yield over the period t to $t+T$. Then, to avoid arbitrage opportunities, $\psi_{t,T}$ must satisfy:

$$\psi_{t,T} = (1 + r_T)P_t - F_{t,T} + k_T \quad (6)$$

where P_t is the spot price at time t , $F_{t,T}$ is the futures price for delivery at time $t+T$, r_T is the risk-free T -period interest rate, and k_T is the per-unit cost of physical storage.

To see why eqn. (6) must hold, note that the (stochastic) return from holding a unit of the commodity from t to $t+T$ is $\psi_{t,T} + (P_{t+T} - P_t) - k_T$. Suppose that one also shorts a futures contract at time t . The return on this futures contract is $F_{t,T} - F_{T,T} = F_{t,T} - P_{t+T}$, so one would receive a total return by the end of the period that is equal to $\psi_{t,T} + F_{t,T} - P_t - k_T$. No outlay is required for the futures contract, and this total return is non-stochastic, so it must equal the risk-free rate times the cash outlay for the commodity, i.e., $r_T P_t$, from which eqn. (6) follows.⁶

Note that the convenience yield obtained from holding a commodity is very much like the dividend obtained from holding a company's stock. (The ratio of the net convenience yield to the spot price, $(\psi_{t,T} - k_T)/P_t$, is referred to as the *percentage net basis*, and is analogous to the dividend yield on a stock.) In fact, if storage is always positive, one can view the spot price of a commodity as the present value of the expected future flow of convenience yield, just as the price of a stock can be viewed as the present value of the expected future flow of dividends.⁷

4.3. Backwardation

From eqn. (6), we can see that the futures price could be greater or less than the spot price, depending on the magnitude of the net (of storage costs) marginal convenience

⁶ One might argue that some cash outlay is required for the futures contract, namely a brokerage firm's margin requirement. However, margin requirements are small for physical commodities, and in any case can be met with interest-bearing assets such as Treasury bills.

⁷ Pindyck (1993) tests this present value model of rational commodity pricing, and finds that it fits the data well for heating oil, but not as well for copper and gold.

yield, $\psi_{t,T} - k_T$. If marginal convenience yield is large, the spot price will exceed the futures price; in this case we say that the futures market exhibits *strong backwardation*. If net marginal convenience yield is precisely zero, we see from eqn. (6) that the spot price will equal the discounted future price: $P_t = F_{t,T}/(1+r_T)$. If net marginal convenience yield is positive but not large, the spot price will be less than the futures price, but greater than the discounted future price: $F_{t,T} > P_t > F_{t,T}/(1+r_T)$. In this case we say that the futures market exhibits *weak backwardation*. (We say that the futures market is in *contango* when $P_t < F_{t,T}$. Thus contango includes weak backwardation and zero backwardation.)

For an extractive resource commodity like crude oil, we would expect the futures market to exhibit weak or strong backwardation most of the time, and this is indeed the case. The reason is that owning in-ground reserves is equivalent to owning a call option with an exercise price equal to the extraction cost, and with a payoff equal to the spot price of the commodity. If there were no backwardation, producers would have no incentive to exercise this option, and there would be no production (just as the owner of a call option on a non-dividend paying stock would wait to exercise it until just before the option expiration date). If spot price volatility is high, the option to extract and sell the commodity becomes even more valuable, so that production is likely to require strong backwardation in the futures market. This is indeed what we observe (see Figure 7, for example); during periods of high volatility, convenience yield is high, so that the spot price exceeds the futures price.⁸

This “real option” characteristic of an extractive resource creates a second reason for the convenience yield to depend positively on the level of volatility. As explained earlier, convenience yield will increase when volatility increases because greater volatility increases the demand for storage; market participants will need greater inventories to buffer fluctuations in production and consumption. But in addition, greater volatility raises the option value of keeping the resource in the ground, thereby raising the spot price relative to the futures price.

⁸ Litzenberger and Rabinowitz (1995) develop a theoretical model of futures prices that incorporates this option value, and they show that the predictions of the model are supported by spot and futures price data for crude oil. For a discussion of the nature and characteristics of the option to produce, see Dixit and Pindyck (1994).

4.4. The Expected Future Spot Price

We turn next to the relationship between the futures price and the expected future value of the spot price. In general, these two quantities need not be equal. To see why this is so, consider an investment in one unit of the commodity at time t , to be held until $t+T$ and then sold (for P_{t+T}). The total outlay at time t for this investment is P_t . Letting E_t denote the expectation at time t , the expected return on the investment is $E_t(P_{t+T}) - P_t + \psi_{t,T} - k_T$. Because P_{t+T} is not known at time t , this return is risky, and must equal $\rho_T P_t$, where ρ_T is the appropriate risk-adjusted discount rate for the commodity. Thus:

$$E_t(P_{t+T}) - P_t + \psi_{t,T} - k_T = \rho_T P_t \quad (7)$$

From eqn. (6), we have $\psi_{t,T} - k_T = (1+r_T)P_t - F_{t,T}$. Substituting this into eqn. (7) gives us the following relationship between the futures price and the expected future spot price:

$$F_{t,T} = E_t(P_{t+T}) + (r_T - \rho_T)P_t \quad (8)$$

Note from eqn. (8) that the futures price will equal the expected future spot price only if the risk-adjusted discount rate for the commodity is equal to the risk-free rate, i.e., there is no risk premium. For most industrial commodities such as crude oil and oil products, we would expect the spot price to co-vary positively with the overall economy, because strong economic growth creates greater demand, and hence higher prices, for these commodities. (In the context of the Capital Asset Pricing Model, the “betas” for these commodities are positive.) Thus we should expect to see a positive risk premium, and the risk-adjusted discount rate ρ_T should exceed the risk-free interest rate r_T . This means that the futures price should be less than the expected future spot price. Intuitively, holding the commodity alone entails risk, and as a reward for that risk, investors will expect that (on average), over the holding period, the spot price will rise above the current futures price.

This difference between the futures price and the expected future spot price can be significant. For crude oil, estimates of “beta” have been in the range of 0.5 to 1. Given an average annual excess return for the stock market of 9 percent, this would put the annual risk premium $\rho_T - r_T$ at 4.5 to 9.0 percent. Thus a six-month crude oil futures contract should “under-predict” the spot price six months out by around 3 to 4.5 percent.

4.5. Hedging and Basis Risk

Futures markets provide a convenient way for producers and consumers of a commodity to reduce risk. In Section 4.1, I gave the example an industrial consumer of crude oil who is worried about the risk of oil price increases during the coming year. As I explained, the consumer could hedge that risk by taking a long futures position in crude oil, and then continuing to buy oil on an ongoing basis from his usual source. If the price of oil rises (falls), the consumer will pay more (less) for his oil, but will have an offsetting gain (loss) from the futures position. The futures contract need not cover the entire time period of concern to the consumer, because the futures position could be repeatedly rolled over. (As I explained above, the consumer would buy oil normally, and never take delivery on the futures contracts.)

Likewise, an oil producer concerned about the risk of oil price decreases could hedge this risk by taking a short position in oil futures. Any decreases in oil prices would then be offset by gains from the futures position. Not surprisingly, producers of oil and oil products typically hold short futures or forward positions.⁹

It is not always possible to hedge all risk with futures contracts. The most common reason is that the commodity specified in the futures contract is not exactly the same as the commodity or asset that one is trying to hedge. For example, most crude oil futures contracts are based on the price of West Texas intermediate crude, and thus such futures can provide only an imperfect hedge on oil that is produced in a different region and/or is of a different grade. Likewise, there is no futures market for jet fuel, so airlines wishing to hedge their exposure to the price of jet fuel will often use a combination of heating oil and gasoline futures instead. Heating oil and gasoline prices (alone or in combination) are not perfectly correlated with jet fuel prices, however, so while this will hedge much of the risk, it will not hedge all of it.

The remaining (unhedged) risk is called *basis risk*. In a hedging situation, the *basis* is defined as the difference between the spot price of the asset to be hedged and the futures price of the contract used to hedge. If the asset being hedged exactly matches that

⁹ Haushalter (2000) provides data on the use of these instruments by oil and gas producers during 1992 to 1994. He shows that, as we would expect, the extent of hedging by these producers is positively related to the effectiveness of such hedging (e.g., it is inversely related to measures of basis risk) and negatively related to the cost of hedging.

specified in the futures contract, the basis will go to zero when the futures contract expires (so there is no basis risk). If the asset being hedged does not match that specified in the futures contract, the basis will not go to zero when the futures contract expires, so there remains some unhedged risk.

4.6. The Behavior of Convenience Yield

Given a spot price and futures price, we can use eqn. (6) to calculate the net (of storage costs) marginal convenience yield, $\psi_{t,T} - k_T$. With information on storage costs, we can obtain the marginal convenience yield $\psi_{t,T}$. As a practical matter, however, there are issues that arise with respect to measuring the spot price, P_t . Although data for futures prices are available daily (or even more frequently), that is not the case for spot prices, i.e., prices for immediate delivery.

Data do exist for *cash prices*, which are actual transaction prices. However, cash prices often do not pertain to the same specification for the commodity (e.g., the same grade of oil at the same location) as the futures price. Also, while cash price data purportedly reflect actual transactions, they usually represent only average prices over a week or a month, and thus cannot be matched with futures prices for specific days. In addition, cash prices usually include discounts and premiums that result from longstanding relationships between buyers and sellers, and thus are not directly comparable to futures prices.

When possible, one can use the price on the *spot futures contract* – i.e., the contract for delivery in the current month – as a proxy for the spot price. However, for most commodities, a spot contract is not available in every month. As a result, to estimate convenience yields, one must often infer a spot price from the nearest and next-to-nearest active futures contracts. This can be done on a daily basis by extrapolating the spread between these contracts backwards to the spot month as follows:

$$P_t = F1_t (F1_t / F2_t)^{n_{0t}/n_1} \quad (9)$$

where $F1_t$ and $F2_t$ are the prices on the nearest and next-to-nearest futures contracts at time t , n_{0t} is the number of days between from t to the expiration of the first contract, and n_1 is the number of days between the first and second contracts.

Using this approach, I have inferred values of the daily spot prices for crude oil, heating oil, and gasoline. Then, using eqn. (6) and the three-month futures price, I computed the net marginal convenience yield, $\psi_{i,T} - k_T$, and I divided these numbers by three to put them in monthly terms. Finally, I estimated the monthly storage cost as the largest negative value of the monthly net marginal convenience yield, and added this in to obtain a series for the monthly marginal convenience yield. Figure 8 shows weekly values over the period January 1984 through January 2001 for the inferred spot price, the three-month futures price, and the corresponding monthly marginal convenience yield for crude oil. Figures 9 and 10 show the corresponding data for heating oil and gasoline.

There are several things that we can observe from these figures:

- For all three commodities, marginal convenience yield fluctuates considerably over time. Some of these fluctuations are predictable, in that they correspond to seasonal variations in the demand for storage. (Other things equal, the demand for heating oil storage, and thus convenience yield, is high during the fall and early winter, and low during the spring and summer. For gasoline, the demand for storage is highest in the late spring and summer, and lowest during the winter.) But much of the variation in convenience yield is unpredictable, and corresponds to unpredictable temporary fluctuations in demand or supply in the cash market.
- Much of the time we observe weak backwardation in the futures markets, but there are frequent and extended periods of strong backwardation, i.e., periods when net marginal convenience yield is sufficiently high that the spot price exceeds the futures price.
- Convenience yield is economically significant. In the case of crude oil, for example, the sample mean of monthly marginal convenience yield is \$0.98 and for the spot price is \$21, which means that on average, market participants are paying nearly 4 percent per month for the privilege of storing oil. For heating oil and gasoline, the mean monthly marginal convenience yields 3.4 percent and 8.1 percent, respectively, of the mean spot price.
- Convenience yield and the spot price are positively correlated, i.e., convenience yield tends to be high during periods when the spot price is unusually high. This does not simply follow from the fact that convenience yield is calculated from the

spread between the spot price and the futures price, because the spot and futures prices also tend to move together. Rather, it follows from the fact that in a competitive market, the spot price is expected to “track,” or slowly revert, to long-run marginal cost. Thus when the spot price is unusually high, it is often the result of temporary shifts in demand or supply that make short-run marginal cost higher than long-run marginal cost. In this situation, inventories will be in high demand because they can be used to reallocate production across time and thereby reduce production costs.

- Although convenience yield and the spot price are positively correlated, they are not perfectly correlated: there are periods when the spot price is unusually high and convenience yield is low, and vice versa. This just reflects the fact that the demand for storage curve can shift, irrespective of shifts of demand and supply in the cash market.

As discussed earlier, the demand for storage, and thus convenience yield, depends in part on the volatility of price changes. We turn next to the behavior of volatility, and its implications for modeling the price process.

5. Volatility and Price Evolution

As we will see, commodity price volatility changes considerably over time, so it is important to understand its behavior. As we have seen, changes in volatility can affect market variables such as production, inventories, and prices. In addition, volatility is a key determinant of the value of commodity-based contingent claims, including financial derivatives (such as futures contracts and options on futures), and real options (such as undeveloped oil and gas reserves).

5.1. The Behavior of Volatility

Using daily data on inferred spot prices, I estimated a weekly series for the standard deviation of daily percentage price changes for crude oil, heating oil, and gasoline. These estimates are essentially a five-week moving sample standard deviation of daily log price changes, for the current week and the previous four weeks, and corrected for non-trading days. (For details of the calculation of these estimates and the correction for non-trading

days, see Pindyck (2001).) I then multiplied the estimate of the standard deviation for each week by $\sqrt{30}$ to put in monthly terms.

Figure 11 shows these price volatility series for each of the three commodities. Two things stand out in this figure:

- First, note that volatility fluctuates dramatically over time. Although the standard deviation of monthly percentage price changes is usually below 10 percent, there have been many occasions when it exceeded 20 percent, and a few occasions when it reached 40 percent or more.
- Second, the volatility series for the three commodities are strongly correlated. (The simple correlation coefficients are .733 for crude oil and heating oil, .732 for crude oil and gasoline, and .767 for heating oil and gasoline.) These correlations are not surprising given that crude oil is a large component of production cost for heating oil and gasoline, so that fluctuations in the price of crude oil will result in corresponding fluctuations in heating oil and gasoline prices.
- Third, fluctuations in volatility are for the most part very transitory. With only a few exceptions, sharp increases in volatility do not persist for more than a month or two. Thus fluctuations in volatility are likely to be important if our concern is with market dynamics in the short run (which so far has been our focus in this paper), but may be less important for longer-run dynamics.

Sometimes there are easily identifiable factors that can explain at least part of the increase in volatility. For example, the volatilities of crude oil, heating oil, and gasoline prices all increased sharply in 1986, and this was due to the sharp decreases in crude prices and the increased uncertainty over the future of OPEC that resulted from Saudi Arabia's decision to vastly increase its production. Likewise, as we discussed earlier, the increase in volatility that occurred in 1990—1991 was due to the Iraqi invasion of Kuwait and the ensuing Gulf War. At other times, however, the causes of increased volatility are much less clear, as are the movements in prices themselves. Whether or not we can explain observed changes in volatility after the fact, it should be clear that these changes are partly unpredictable. Indeed, Figure 11 suggests that volatility follows a rapidly mean-reverting stochastic process.

5.2. Modeling the Underlying Price Process

When evaluating commodity-based investment projects and other contingent claims, it is often assumed that the price of the commodity follows a geometric Brownian motion (GBM).¹⁰ In other words, future values of the logarithm of price are assumed to be normally distributed, with a variance that increases linearly with the time horizon. This is analytically convenient because it yields relatively simple solutions.¹¹ It assumes, however, that volatility is a fixed parameter, and as Figure 11 shows, this hardly seems to be the case. Figure 11, along with the price series shown in Figures 8, 9, and 10, suggest that the spot price might be better represented by some kind of mean-reverting process.

Given that we expect the spot price to revert to long-run marginal cost, which itself may drift randomly due to technical change and changing expectations regarding the reserve base, it would be logical to model price evolution by separating fluctuations in the short run from those in the long run. Schwartz (1997) and Schwartz and Smith (2000) have specified and estimated models that do just this: The equilibrium price level is assumed to follow a GBM, while short-term deviations from the equilibrium price are assumed to revert toward zero following an Ornstein-Uhlenbeck process.¹² These models seem to fit the data well, and help to explain the short run behavior of volatility and marginal convenience yield. Their use, however, considerably complicates the valuation of financial and real options.

Lo and Wang (1995) calculated call option values for stocks with prices that follow a trending Ornstein-Uhlenbeck process, and compared these to the values obtained from the Black-Scholes model (which is based on a GBM for the stock price). They showed that the Black-Scholes model can over- or underestimate the correct option value, but generally the size of the error is small, at least relative to errors that would be tolerable in real option applications. (They find errors on the order of 5 percent of the option value,

¹⁰ A good example of this is Paddock, Siegel, and Smith (1988).

¹¹ When the price process is a GBM, it is written as $dP = \alpha P dt + \sigma P dz$, where dz is the increment of a Weiner process (i.e., $dz = \varepsilon(dt)^{1/2}$, where ε is normally distributed and serially uncorrelated. See Dixit and Pindyck (1994) for details.

¹² If the log of price follows an Ornstein-Uhlenbeck (mean-reverting) process, we would write it as $dp = \lambda(p^* - p)dt + \sigma dz$, where $p = \log(P)$ and p^* is the long-term price to which p reverts. The estimation of this and related processes is discussed in, Campbell, Lo, and MacKinlay (1997).

which would be significant for a financial option. In the case of a capital investment decision, there are enough other uncertainties regarding the modeling of cash flows that an error of this size is unlikely to be important.)

Furthermore, financial options typically have lifetimes of a few months to a year, while real options are much longer lived. If one is concerned with investment decisions such as the development of oil reserves or the construction of power plants, pipelines, or refineries, the long-run behavior of prices and volatility is more relevant. Using price series of more than a century in length, Pindyck (1999) has shown that the average growth rates of the real prices of oil, coal, and natural gas have been quite stable (and close to zero) over periods of 20 to 40 years, as have the sample standard deviations of log price changes. Over the long run, price behavior seems consistent with a model of slow mean reversion. If this is indeed the case, for investment decisions in which energy prices are the key stochastic state variables, the GBM assumption is unlikely to lead to large errors in the optimal investment rule.

6. Conclusions

We have seen how commodity spot prices, futures prices, and changes in inventories are determined by the equilibration of demand and supply in two interrelated markets – a cash market for spot sales, and a market for storage. I have described the characteristics of supply and demand in these two markets, shown how supply and demand respond to external shocks, and explained how these markets are connected to futures prices and the futures-spot spread. As the title of this article indicates, my objective has been to provide a “primer,” so the discussion has necessarily been brief. The interested reader can find more detailed and rigorous discussions of commodity market dynamics elsewhere. (See, for example, Routledge, Seppi, and Spatt (2000) and Williams and Wright (1991).)

In principle, one could specify and estimate equations for demand and supply in the cash and storage models, and thereby forecast the effects of volatility shocks, changes in the weather, or other shocks on the short-run behavior of prices and other variables. Models of this kind have been estimated by Pindyck (2001) using the weekly data that have been presented here. These models provide good out-of-sample forecasts of spot prices and convenience yields for periods of 4 to 8 weeks.

In this paper, I have provided an explanation of short-run commodity price movements that is based on “fundamentals,” i.e., rational shifts in supply and demand in each of two markets. We might expect that some portion of commodity price variation is not based on fundamentals, but is instead the result of speculative “noise trading” or herd behavior, and there is evidence that this is indeed the case. For example, Roll (1984) found that only a small fraction of price variation for frozen orange juice can be explained by fundamental variables such as the weather, which in principle should explain a good deal of the variation. And Pindyck and Rotemberg (1990) found high levels of unexplained price correlation across commodities that are inconsistent with prices that are driven solely by fundamentals.

Such findings do not invalidate the basic model of short-run commodity market dynamics that has been presented here. In fact, we can incorporate speculative behavior in the error terms of the model. For example, speculative holdings of inventories can be included in the error term ε_3 in eqn. (5) for the demand for storage. Most important, our model of “fundamentals” can explain a large part of the short-run dynamics of prices and other variables, and can help us understand how commodity markets respond to changes in various exogenous variables.

Appendix: Definitions of Key Terms

This appendix provides definitions of some the key terms used in the paper. The definitions provided here are brief; for more detailed explanations of the terms, the reader should refer to a text on commodity futures markets.

Basis Risk: The risk that results when the settlement price of a hedging instrument differs from the price of the underlying asset being hedged. For example, most crude oil futures are based on the price of West Texas intermediate crude and thus such futures can provide only an imperfect hedge on oil that is produced in a different region and is of a different grade.

Cash Price: An average transaction price, usually averaged over a week or a month. May also include discounts or premiums that result from longstanding relationships between buyers and sellers, and thus is not equivalent to the spot price.

Commodity: A homogeneous product. Many products are not perfectly homogeneous, but are sufficiently so to be viewed as commodities; crude oil and oil products are examples.

Contango: Condition in which the spot price is less than the futures price.

Convenience Yield: The flow of benefits to the holder of commodity inventory. These benefits arise from the use of inventories to reduce production and marketing costs, and to avoid stockouts. **Marginal convenience yield** is the flow of benefits accruing from the marginal unit of inventory, and is thus equal to the **price of storage**.

Cost of Carry: A portion of the total cost of storing a commodity, namely the physical storage cost plus the forgone interest.

Forward Price: The price for delivery at some specified future date, to be paid at the time of delivery. The price, quantity, and commodity specifications are spelled out in the **forward contract**.

Futures Price: The price for delivery as some specified future date, with terms and conditions specified in the **futures contract**. Payments, however, are made at the end of each trading day as the futures price changes. Thus futures contracts are **marked-to-market**, which means there is a “settling up” between buyers and sellers at the end of each trading day.

Net Demand: The demand for production in excess of consumption.

Net Marginal Convenience Yield: Marginal convenience yield minus the cost of physical storage.

Operating Options: Options to produce now (at an “exercise price” equal to marginal production cost and with a “payoff” equal to the spot price), rather than waiting for possible increases or decreases in price that might occur in the future.

Percentage Net Basis: The ratio of the net (of storage costs) convenience yield to the spot price, $(\psi_{t,T} - k_T)/P_t$.

Settlement Price: The market-clearing futures price at the end of the trading day. On an actively traded contract, it is the last price at or very close to the close of trading; otherwise it is an estimate by the futures exchange of the market-clearing price at the end of trading.

Spot Price: Price for immediate delivery of a commodity.

Spot Contract: Futures (or forward) contract for delivery in the current month. A spot contract may not exist or may not be actively traded for every commodity in every month.

Storage Cost: The cost of physical storage for a commodity.

Strong Backwardation: Condition in which the spot price exceeds the futures price.

Weak Backwardation: Condition in which the spot price is less than the futures price, but exceeds the *discounted* futures price: $F_{t,T} > P_t > F_{t,T}/(1+r_T)$.

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FIGURE 1: EQUILIBRIUM IN THE CASH MARKET

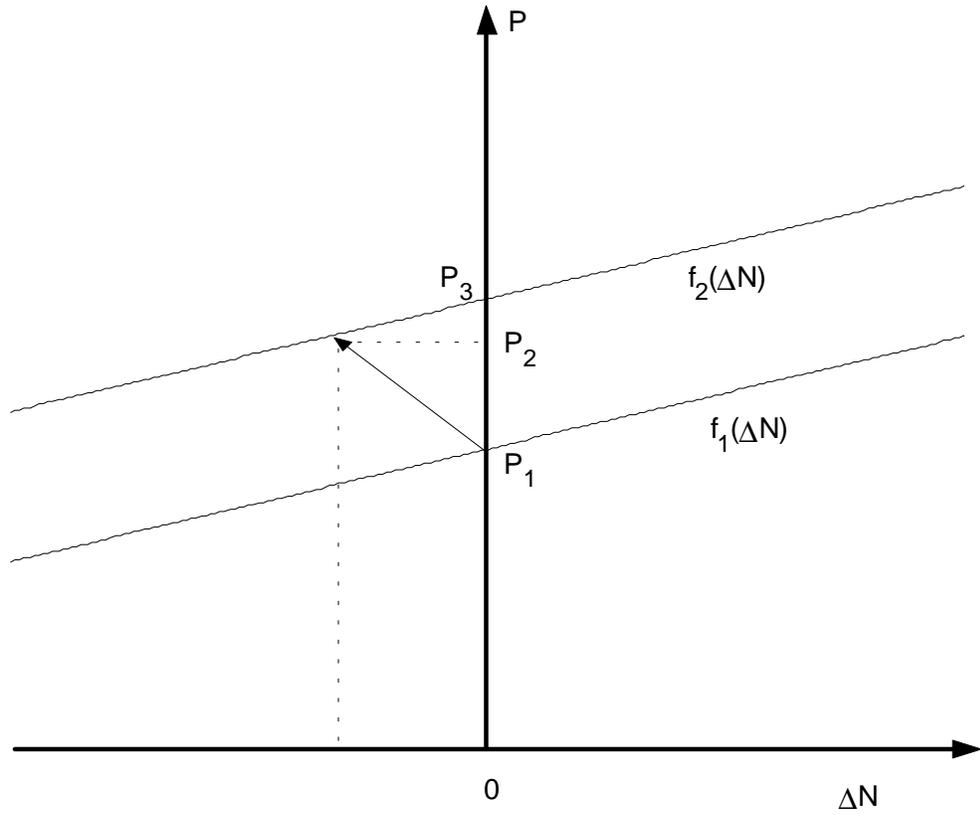


FIGURE 2: EQUILIBRIUM IN THE STORAGE MARKET

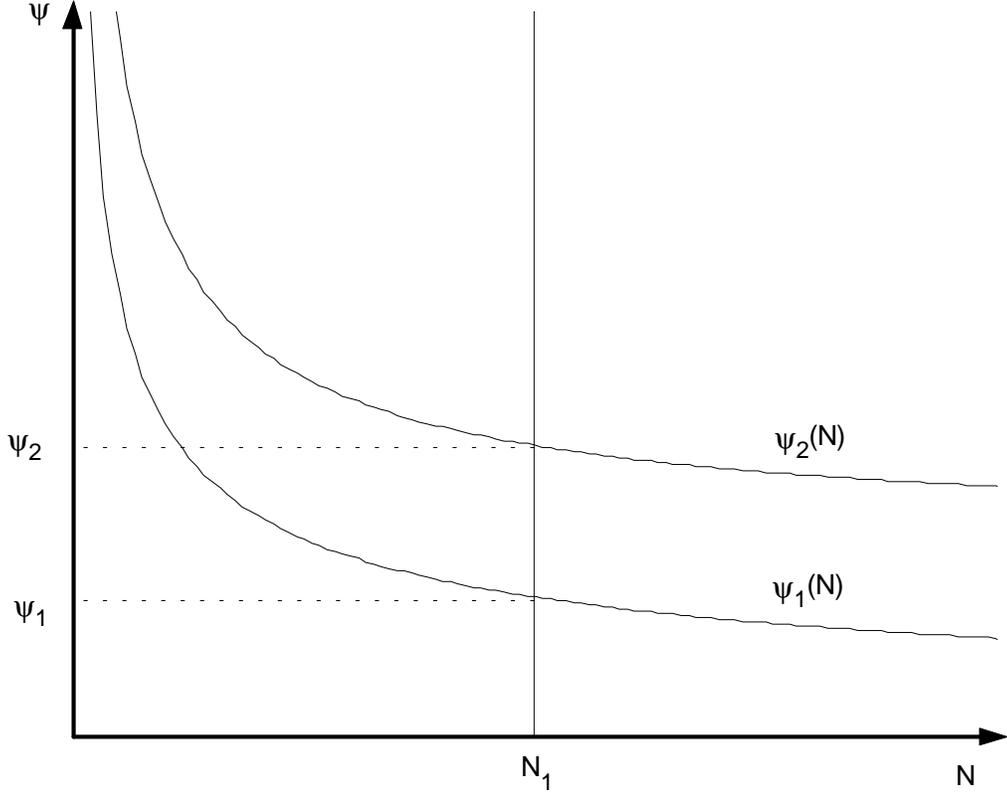


FIGURE 3: EFFECT OF TEMPORARY DEMAND SHOCK

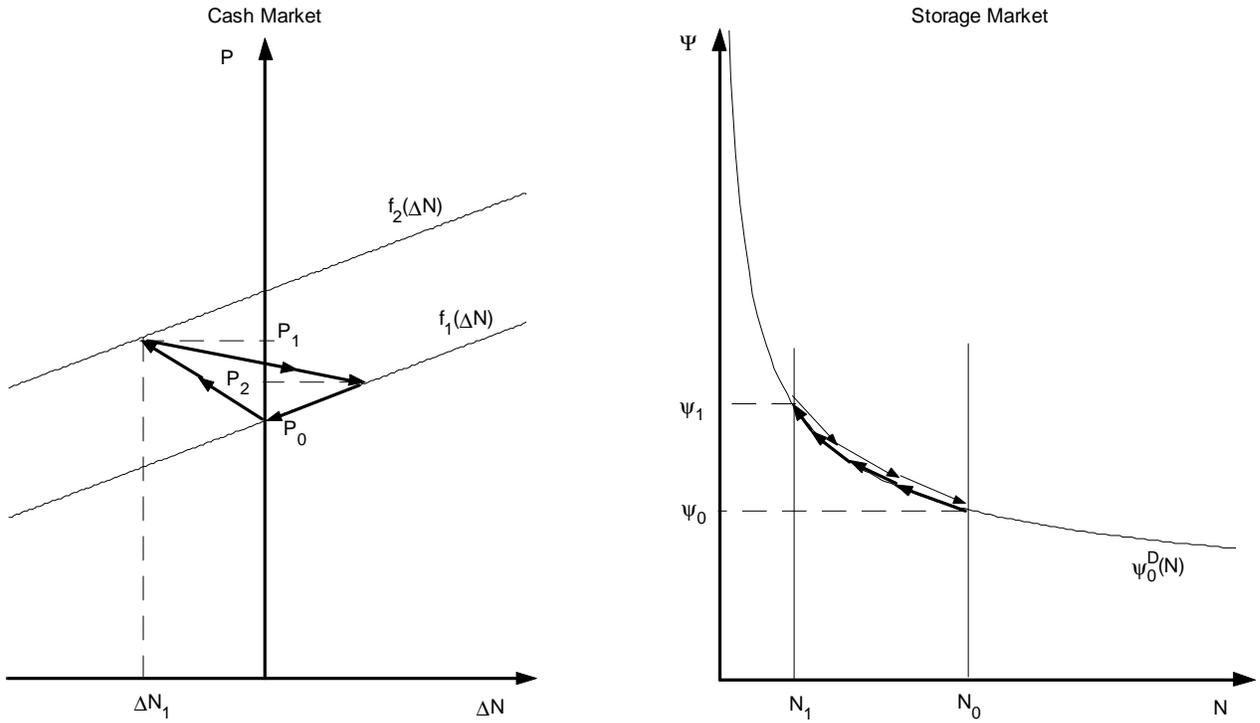


FIGURE 4

PRICES OF HEATING OIL AND CRUDE OIL

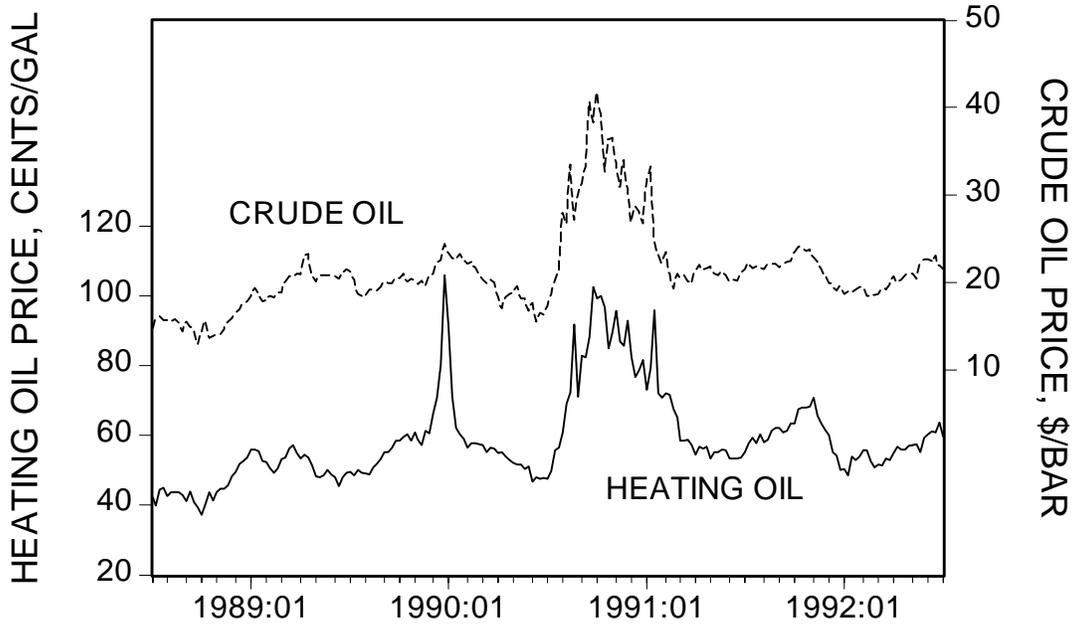


FIGURE 5

HEATING OIL CONVENIENCE YIELD AND HEATING DEGREE DAYS

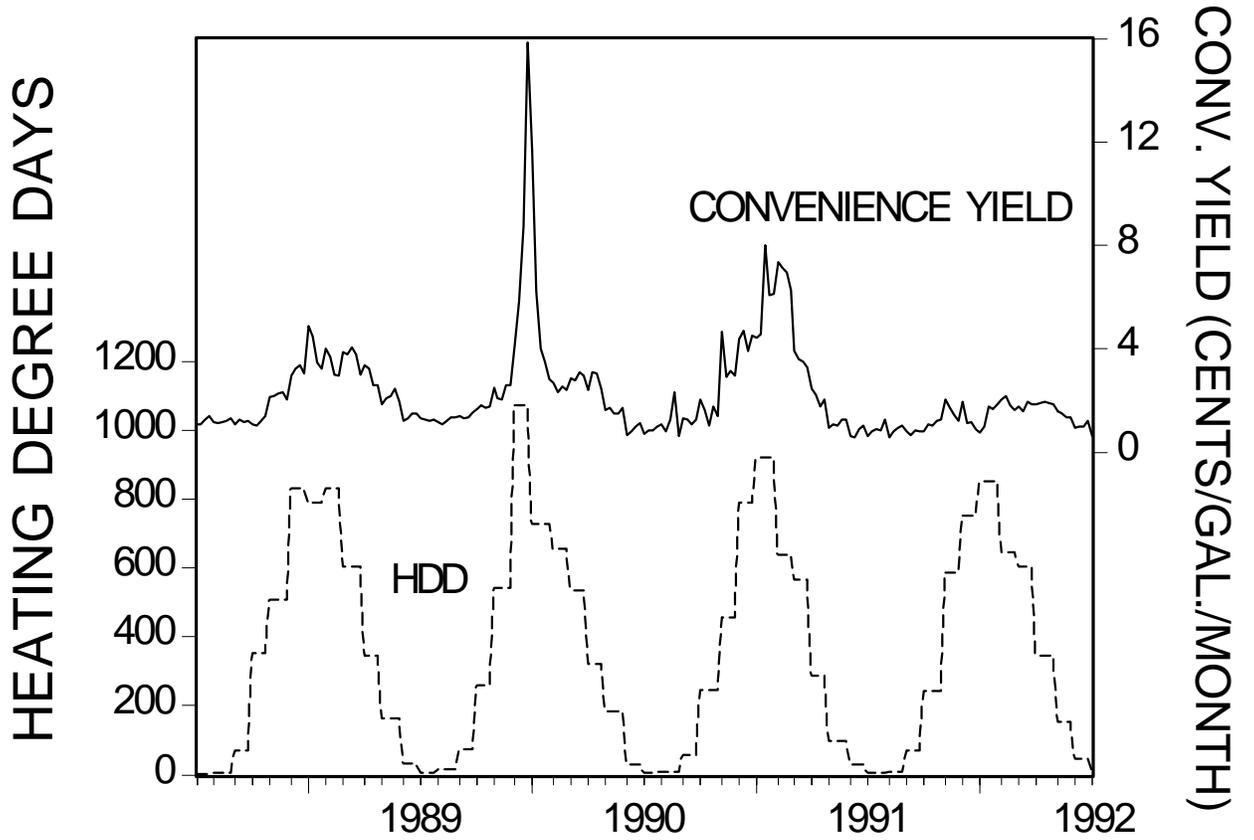


FIGURE 6: RESPONSE TO INCREASE IN VOLATILITY

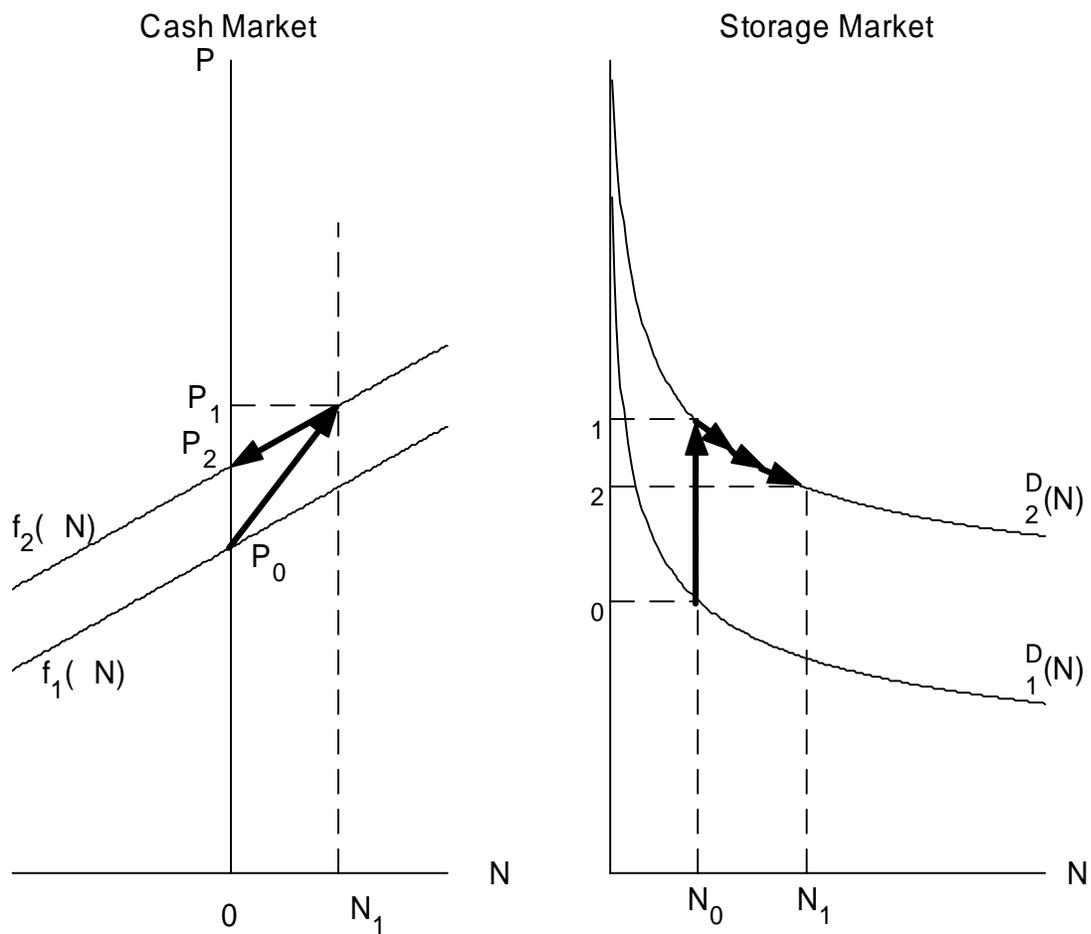


FIGURE 7

CRUDE OIL -- VOLATILITY, SPOT PRICE,
AND CONVENIENCE YIELD

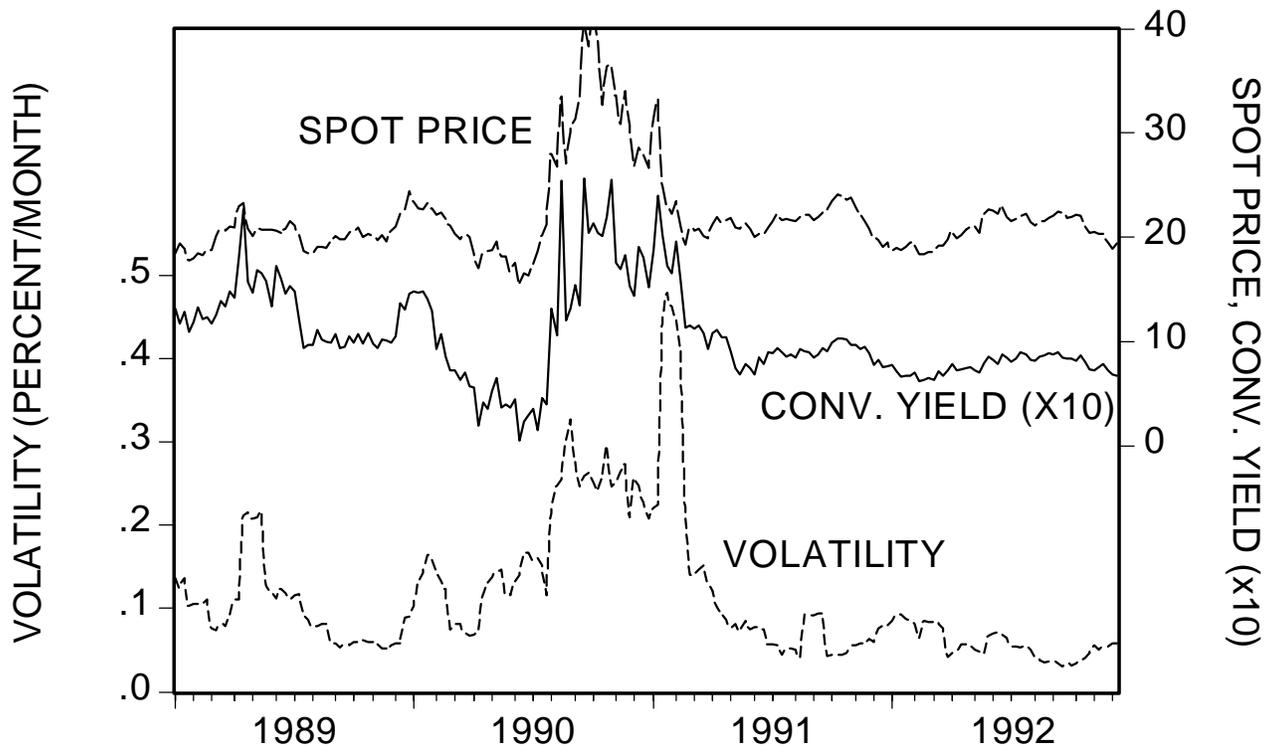


FIGURE 8

CRUDE OIL -- SPOT PRICE, FUTURES PRICE,
AND CONVENIENCE YIELD

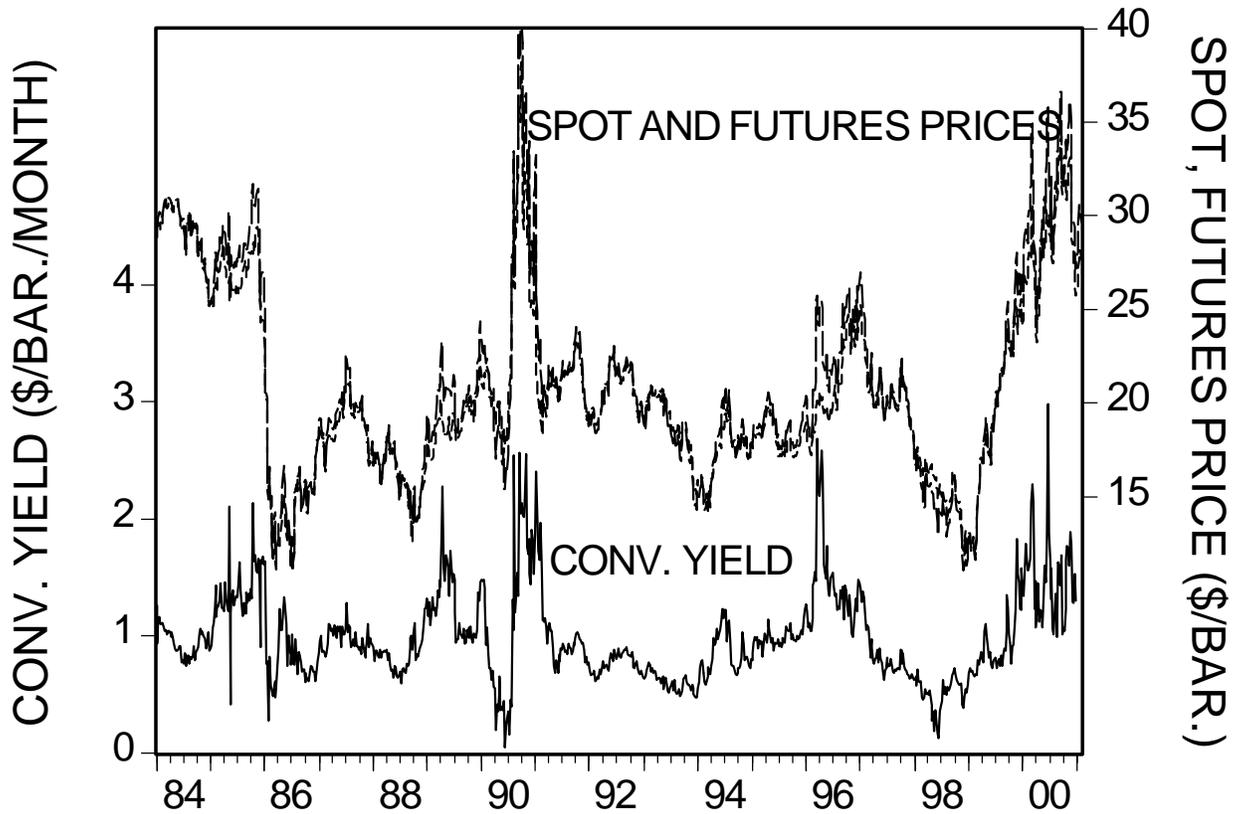


FIGURE 9

HEATING OIL -- SPOT PRICE, FUTURES PRICE,
AND CONVENIENCE YIELD

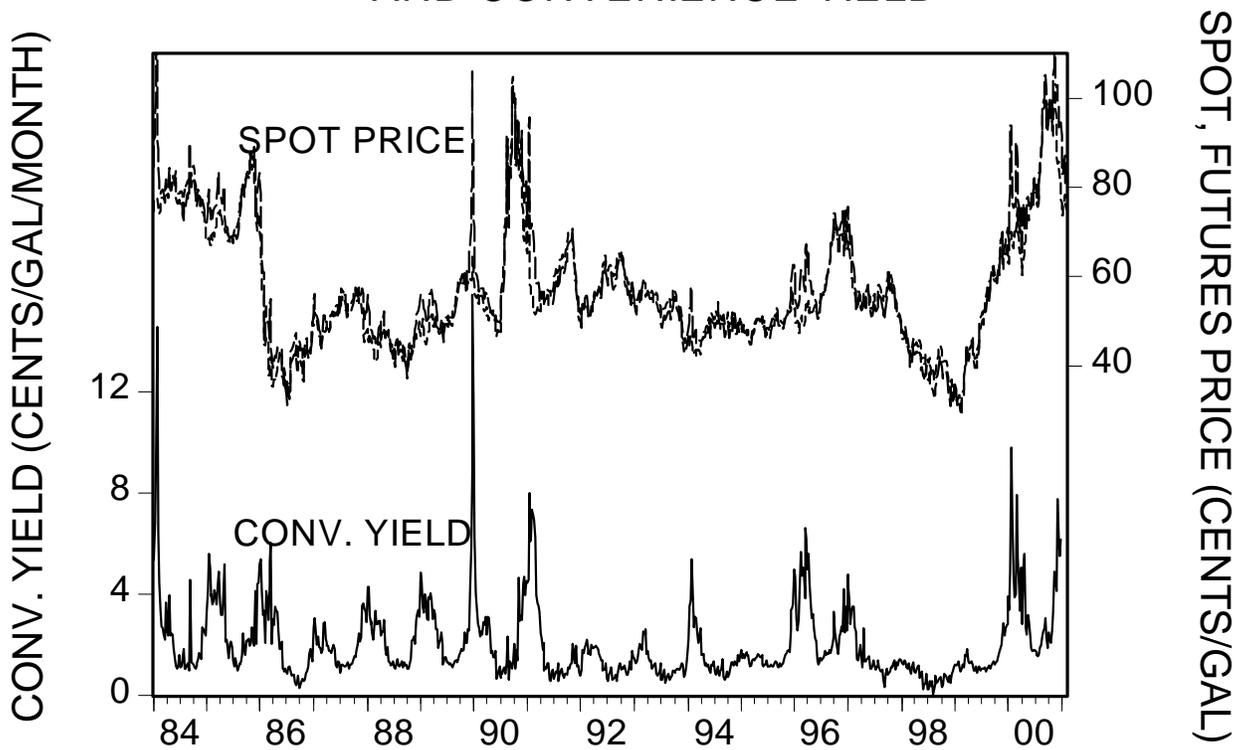


FIGURE 10

GASOLINE -- SPOT PRICE, FUTURES PRICE,
AND CONVENIENCE YIELD

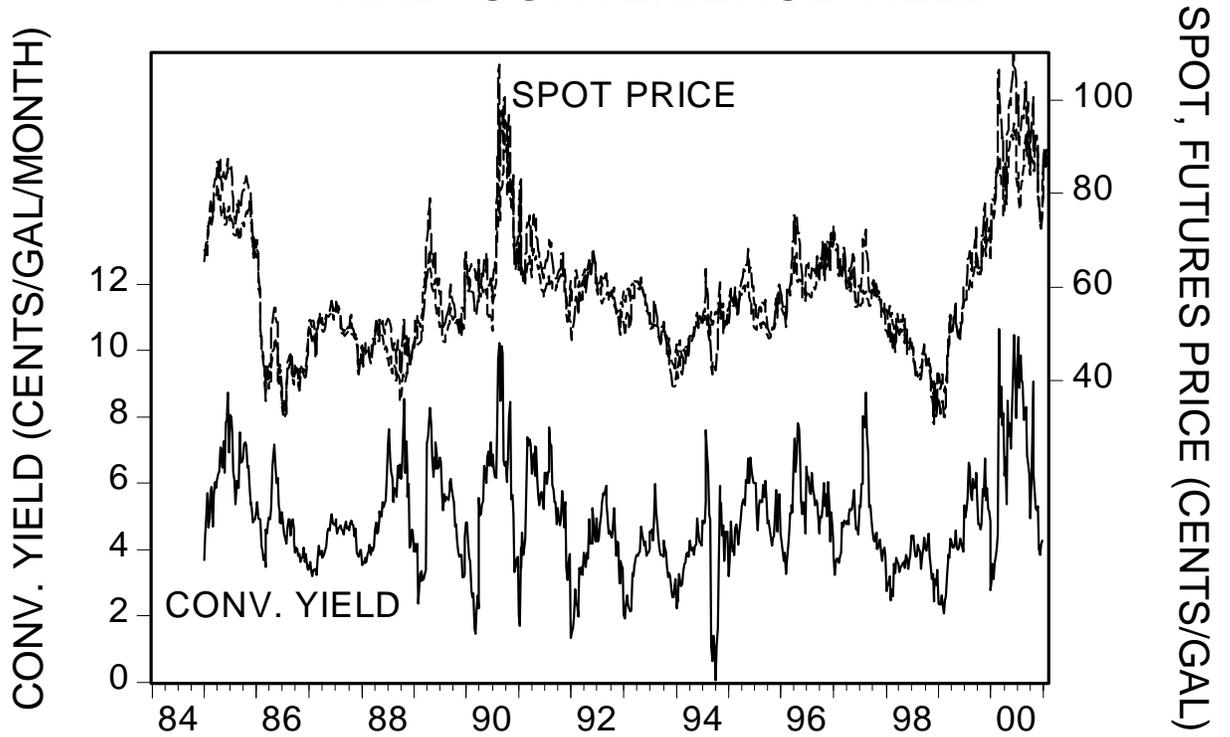


FIGURE 11

STANDARD DEVIATION OF MONTHLY PERCENTAGE PRICE CHANGES -- CRUDE, HEATING OIL, GASOLINE

