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Population, Productivity, and Sustainable Consumption

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POPULATION, PRODUCTIVITY, AND SUSTAINABLE CONSUMPTION*

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Abstract: How does the sustainable level of consumption depend on productivity growth and the size and growth rate of the population? What is the effect of uncertainty over these growth rates? I address these questions using a model in which productivity and population growth are stochastic, and social welfare allows for human lives to have (positive or negative) intrinsic value. I show how the maximum sustainable consumption-wealth ratio depends on expected rates of productivity and population growth, volatility of those growth rates, and the extent to which welfare depends directly on the size of the population. For plausible parameter values, the sustainable consumption-wealth ratio is well below the optimal ratio that maximizes welfare. This raises a question: Given its cost, should sustainability be a social objective?

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1 Introduction.

Given current and projected future levels and growth rates of aggregate production and wealth, what level of consumption is sustainable? How does the sustainable level of consumption depend on the size and growth rate of the population? How does it compare to the optimal level of consumption that maximizes welfare? And how are the answers to these questions affected by uncertainty, over both the growth and productivity of the capital stock and the growth of population?

These questions presume a definition of "sustainable." A common definition is that future generations should be at least as well off as we are. But does "as well off" mean there is no reduction in per-capita consumption, or no reduction in the utility from consumption? And do we care about the number of people who are well off?

Much of the economics literature that addresses these questions defines a sustainable path for consumption as one for which social welfare is non-declining throughout the future. In turn, social welfare is usually defined as the present value of a flow of utility generated from consumption, which can be broadly measured to include goods and services, but also the value of leisure, health, and environmental amenities. Social welfare is then:

$$V_t = \int_t^{\infty} U(C_\tau) e^{-\rho(\tau - t)} d\tau \ , \tag{1}$$

where ρ is the social rate of time prefence (and hence discount rate) and sustainability boils down to the requirement that $dV/dt \geq 0$ for all t.

A variety of studies have used this framework to examine how sustainable consumption can depend on such things as the level and growth rate of the capital stock, depletion of natural resources, and technological change. For example, Dasgupta (2009) and Arrow et al. (2012, 2013) show that sustainability implies that a properly defined comprehensive measure of productive wealth — which includes stocks of physical and human capital, natural resources, and the technological knowledge base — must never decline. Others have used this framework to assess whether current consumption exceeds the sustainable level.[1]

The studies cited above and others like them have two important limitations. First, they

[1]For an overview of studies of sustainable consumption, see Arrow et al. (2004). As they point out, a sustainable trajectory for consumption may not exist, and if it does it may not be unique and it need not be optimal (in the sense of maximizing V_t). Are we consuming too much? Arrow et al. (2004) answer this by measuring the growth rates of per capita "genuine wealth," (i.e., comprehensive productive wealth) for different countries. They show that some (mostly poor) countries are on unsustainable trajectories because their investments in physical and human capital do not offset their depletion of natural capital.

are inherently deterministic in nature. They typically examine how sustainable trajectories for consumption depend on the (deterministic) growth rates of the capital stock, productivity, natural resources, and other factors that affect output and welfare. These studies yield insights into the relative importance of different factors that can limit future consumption, but they ignore the fact that the economy evolves stochastically, so it is impossible to ensure that welfare will never decline. An important exception to this literature is the fully stochastic model developed by Campbell and Martin (2021), discussed below.[2]

A second limitation is that population is usually taken as incidental. Eqn. (1) might be modified by replacing C with per capita consumption C/N, with population N growing at some exogenous rate. But there is no social utility (or disutility) from the very existence of people. This is at odds with a growing literature that examines how life itself might be valued. There are good reasons to believe that population growth will affect social welfare (apart from its impact on per-capita consumption), and we will see that this can have profound implications for sustainable consumption.

In a world where the determinants of welfare, and thus welfare itself, evolve stochastically, sustainability can be defined in expected value terms. A natural definition is that the expected value of social welfare (which itself evolves stochastically) is not expected to decline at any point in the future. That is the definition of sustainability used in the recent paper by Campbell and Martin (2021) (hereafter CM) on which this paper builds, and is used here as well. With this definition of sustainability, eqn. (1) becomes

$$V_t = \mathcal{E}_t \int_t^{\infty} U(C_\tau) e^{-\rho(\tau - t)} d\tau \,, \tag{2}$$

and sustainability requires $(1/dt)\, \mathcal{E}_t \, dV \geq 0$ for all t.

In their paper CM assume that social welfare derives from wealth-generated consumption via a constant relative risk aversion (CRRA) utility function, that wealth can be invested in risk-free and risky capital, and that the value of risky capital is driven by both Brownian motion and Poisson jumps of random size. They show how these two types of stochastic shocks affect portfolio choice, i.e., the fraction of wealth society optimally invests in risky capital, and the sustainable consumption-wealth ratio. They find that the sustainability constraint does not affect portfolio choice, and the sustainable consumption-wealth ratio lies between the risk-free interest rate and the expected return on optimally invested wealth.

[2] Agliardi (2011) shows how the stochastic evolution of the capital stock and productivity can affect the expected change of V_t, but does not solve for V_t.

A definition of sustainable consumption based on the expected rate of change of expected future social welfare should not be controversial. But how should we define social welfare? CM take the standard approach by defining social welfare as the expected discounted flow of CRRA utility from consumption. I use parts of the CM framework here (sticking to their notation where possible to facilitate comparisons with their paper), but I use a more general definition of social welfare. In particular, I include population in the social welfare function, and allow both population and productive wealth to evolve stochastically.

How might population affect social welfare? I consider the following:

1. Total productive wealth generates total consumption, but what matters for individual welfare is per-capita consumption. Holding total consumption fixed, when population increases, per-capita consumption falls. If the population is homogeneous, the welfare of each individual falls, and thus so does social welfare. (CM also allow for this possibility.)

2. People consume but they also produce, so total consumption should depend on the total population. If the population increases, does the productive capacity of the economy, and hence total output and consumption, increase more or less than proportionally in response? Although the subject of considerable research, to my knowledge there is no clear answer to that question. Thus I examine how sustainable consumption depends on how population growth affects aggregate production.

3. One could argue (as both economists and philosophers have) that society values the very existence of people — alive now and potentially in the future — and not just the consumption enjoyed by those people and/or their contribution to aggregate production. Then population can enter the social welfare function in a more complex way, as discussed below.

I start with a simplified version of the CM model. Like CM: (1) I assume that all individuals are the same, i.e., there is no heterogeneity within the population. (2) I assume that production and hence consumption requires productive wealth, which includes physical and human capital, as well as the technological know-how to make that capital productive. (3) I measure sustainable consumption in terms of its relationship to wealth, i.e., I calculate a sustainable consumption-wealth ratio, and compare it to the optimal (unconstrained) ratio that maximizes welfare. But unlike CM, I assume that all productive wealth is risky, so I can ignore portfolio choice. I also assume that all fluctuations in wealth are continuous (there are no jumps). Eliminating portfolio choice and Poisson jumps greatly simplifies the analysis, and facilitates the introduction of population as a second stochastic state variable.[3]

My model diverges from CM in three other important respects. First, I assume production requires labor, and I take the labor force to be proportional to population.[4] I assume that the relationship between population and output is isoelastic, and I examine how the elasticity affects the sustainable consumption-wealth ratio. Second, I introduce a more general social welfare function that explicitly includes population, so I can explore how sustainable consumption depends on the extent to which we value the existence of people, apart from their consumption and their contribution to aggregate output. And third, I assume that population evolves as a continuous stochastic process.

Because I assume that productive wealth and population follow geometric Brownian motions, social welfare in my model also follows a geometric Brownian motion, and its drift depends on the consumption-wealth ratio. I define a sustainable consumption-wealth ratio as one for which the drift of the stochastic process for social welfare is non-negative. (Hence the expected value of social welfare is not expected to decline at any point in the future.) This condition yields a constraint on the maximum consumption-wealth ratio. In addition, the social welfare function can be determined analytically, and then maximized to yield the optimal (unconstrained) consumption-wealth ratio.

The model, presented in the next section, yields several insights: (1) As in earlier deterministic models, if the return on capital is low and/or population growth is high, a positive sustainable consumption-wealth ratio may not exist. (2) An increase in the volatility of the return on capital always reduces the sustainable consumption-wealth ratio, but an increase in the volatility of population growth can increase or decrease the ratio, depending on the parameters of the model. (3) Sustainable consumption depends critically on the extent to which lives have intrinsic social value. A positive (negative) intrinsic social value of lives raises (reduces) the sustainable consumption-wealth ratio. The reason is that consumption and population become substitutes in terms of their contributions to social welfare. (4) For plausible parameter values, the sustainable consumption-wealth ratio is well below the optimal ratio that maximizes social welfare. This implies that achieving sustainability can come at the cost of a substantial welfare loss.

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[3]CM also show how their results change when there is no risk-free asset and there are no jumps in wealth.

[4]Arrow, Dasgupta and Mäler (2003) also examine sustainable consumption when welfare depends on per-capita consumption and population (labor) also enters the production function, but in a completely deterministic setting.

2 The Model.

I characterize social welfare in terms of the following utility function:

$$U(C,N) = \frac{1}{1-\gamma}\left[(C/N)^{1-\phi}N^{\phi}\right]^{1-\gamma} \tag{3}$$

This is a CRRA utility function, not of consumption alone but rather of a composite that combines per-capita consumption (C/N) and population N, with weights $1-\phi$ and ϕ respectively, and $\phi < 1$. If $\phi = 0$, social welfare depends only on per-capita consumption, as in most models of sustainability. But if $\phi \neq 0$, eqn. (3) says that we (that is, society) care not only about our individual consumption, but also about the very existence of other people. Depending on the value of ϕ, holding per capita consumption fixed, we might prefer the existence of more people to fewer people, or the reverse.

Utility functions related to (3) have been used by others in different contexts. Ashraf and Galor (2011), for example, developed a Malthusian growth model with technological change in which individual utility depends on consumption and also on the number of surviving children, with weights $1-\phi$ and ϕ respectively, as in (3).[5] But unlike these other studies that are focused on growth, the emphasis here is on the welfare effects of changes in population and productive wealth, and the implications for sustainable consumption.

In what follows, I assume $\gamma > 1$. If $\phi = 0$ so social welfare depends only on per-capita consumption, γ is a coefficient of relative risk aversion, and $\gamma > 1$ would be consistent with economic and financial data. More generally, γ simply defines the marginal social utility of an increase in the $[(C/N), N]$ composite, whether resulting from a change in C, N, or both.

For now I assume that C is independent of N. (This assumption will be relaxed shortly.) With $\phi < 1$ the utility function (3) has the usual dependence on consumption, i.e., $U_C > 0$, $U_{CC} < 0$, and $U_C \to \infty$ as $C \to 0$. However, U_N can be positive or negative, depending on ϕ. If $\phi = 0$, a catastrophic event that causes a drop in N is a pure blessing for those that remain alive, because per-capita consumption rises and N itself is not directly valued.[6] In general, $U_N >$ (<) 0 if $\phi >$ (<) 1/2. In the special case of $\phi = 1/2$, a drop in N yields a drop in welfare that is just offset by the gain from the increase in C/N, so $U_N = 0$.

What can we say about the value of ϕ? It is easy to argue that social welfare should

[5] They assume $\phi \geq 0$. For some variations on (3), see Galor (2011).

[6] This is similar to the "benefits" of wars and major pandemics in the articles by Young (2005) and Voigtländer and Voth (2009, 2013).

be an increasing function of per capita consumption, but should it also be an increasing (or decreasing) function of population? More generally, why might we care at all about the existence of other people, and why should a larger population be more or less preferable to a smaller one? These questions have been addressed from several angles. Studies of the optimal population size or growth rate have used alternative social welfare functions to evaluate outcomes, in the context of growth models in which population can be exogenous or endogenous.[7] The social choice literature uses instead an axiomatic approach that derives social orderings for which population is one of the choice variables (see, e.g., Broome (2004) and Blackorby, Bossert and Donaldson (2005)).

As a practical matter, we tend to value changes in population asymmetrically. Based on the social norms of most countries, mortality-based *decreases* in population are almost always treated as "bad," in that societies go to great lengths to save lives and prevent life-threatening disasters. (But not so for decreases dues to low birth rates.) *Increases* in population, on the other hand, are seen by some as "good" (on purely ethical grounds, but also by contributing to technological change, and ensuring the old are cared for), and by some as "bad" (because of crowding and environmental stress). Thus social welfare should depend at least in part on population, even though there may be no consensus regarding the form of that dependence, and more generally on the value of more (or fewer) people.

Why might an increase in N be bad? A common argument is that more people cause unwanted "congestion," in the broad sense of the term. This argument is usually environmental in nature, e.g., more people will crowd our national parks, accelerate climate change and pollution generally, and more rapidly deplete our natural resources.[8] Even if resources

[7] Welfare functions include the utility of a representative agent, per-capita utility, and the sum of utilities over the existing population. Samuelson (1975) derived an optimal exogenous population growth rate in the context of an overlapping generations growth model. More recent work made population growth endogenous and allowed a parent's utility to depend on both her own consumption and the utility and number of her children; see, e.g., Harford (1998), Razin and Sadka (1995), and Ashraf and Galor (2011). When welfare depends on the utility of those who exist as well as those yet to be born, Pareto efficiency is not well-defined, unless it is based on a representative agent whose preferences apply across generations.

[8] See, e.g., Harford (1998) and Bohn and Stuart (2015). Based on climate change impacts, Casey and Galor (2016) estimate a social benefit from slower population growth. Acemoglu, Fergusson and Johnson (2020) show that increases in population resulting from improvements in health have led to more civil wars and other forms of violent conflict. Finally, is there a limit to the number of people that can live (tolerably) on Earth, and if so what is it? We don't know, but see Cohen (1995), who surveys attempts to estimate the human carrying capacity of the Earth. Lianos and Pseiridis (2016) use measures of the biocapacity of different countries to estimate the maximum population that would support an "acceptable" level of per capita consumption indefinitely. They find a few countries are underpopulated (e.g., Canada, Russia), but most are overpopulated (e.g., China, India).

and externalities are priced properly, welfare can fall. For example, if carbon emissions are taxed at their social cost, an increase in N that increases emissions will still reduce total and per-capita consumption. And if the externalities are not priced properly (as is usually the case), the negative impact of an increase in N would be even greater. Thus environmentalists (and others) might argue for a low or even negative value of ϕ, so that $U_N < 0$.

One argument for a social preference for a *higher* N is that more people are needed to drive technological progress and economic growth, as in Kremer (1993), Jones (1999), Desmet, Nagy and Rossi-Hansberg (2018), and related models.[9] Population growth is also needed to provide the workers to generate output, especially as the pool of older retirees expands. As an empirical matter, many societies exhibit a preference for a higher N, insofar as they try to prevent or limit the deaths of their citizens, and subsidize the bearing and rearing of children.[10] Finally, many will argue that lives simply have intrinsic social value, so that 2 million people each enjoying utility U_0 is preferred to 1 million people enjoying that same utility. This argument implies a social preference for a higher N, and thus a higher value of ϕ so that $U_N > 0$ and $U_{NN} < 0$.

We will explore the implications for sustainable consumption of a social preference for more or fewer people. But first we need to account for the fact that consumption requires production, which requires people.

2.1 Consumption, Wealth, and Population.

Consumption in this model is generated by productive wealth, W_t:

$$C_t = \theta W_t \ ,$$

where θ is the consumption-wealth ratio, and is assumed constant. (We want the maximum value of θ consistent with the sustainability constraint.) Productive wealth W_t is in turn a

[9] As Jones (2021) shows clearly, in semi-endogenous growth models research-generated growth is proportional to the rate of population growth, so declines in population growth could reduce GDP growth.

[10] Societies regularly make decisions that weigh lives against other "good things," such as consumption, and there is a large literature that deals with both the economic and ethical aspects of those trade-offs. See, e.g., Broome (2004) and Blackorby, Bossert and Donaldson (2005). Millner (2013) provides an argument for including population as a determinant of social welfare. The recent innovative paper by de la Croix and Doepke (2021) shows how we might value changes in population.

function of capital (physical and human) and technological know-how, but also population:

$$W_t = A_t N_t^{\beta} \ ,$$

with $\beta \geq 0$. Here A_t reflects the broad-based stock of capital and its productivity. As long as $\beta > 0$, a greater population implies more labor and hence more output and more total consumption. Also, we might have $\beta > 1$, i.e., increasing returns to population growth.

How does W_t change over time? It will increase if A_t increases (as productivity increases and/or some part of output is invested in additional capital) or if N_t increases. It will decrease if A_t or N decrease, but also as a fraction of wealth, θ, is consumed.

Substituting $C_t = \theta A_t N_t^{\beta}$ into (3), the utility function becomes

$$U(A, N) = \frac{1}{1-\gamma} \left[(\theta A_t N_t^{\beta-1})^{1-\phi} N_t^{\phi} \right]^{1-\gamma} \tag{4}$$

If $\beta = 0$, C is independent of N and we are back to utility function (3). If $\beta = 1$, an increase in N leaves $C/N = AN^{\beta-1} = A$ unchanged, and if $\beta > 1$, an increase in N results in an increase in C/N. What is a reasonable value for β? The share of GDP paid to labor is around 2/3, which suggests a value for β around 0.6 or 0.7. This would imply that an increase in N alone would reduce C/N (but might increase welfare, depending on the value of ϕ); with $\beta < 1$, any increase in C/N requires an increase in A, which we treat as exogenous. On the other hand, some studies suggest that β may be greater than 1.[11]

Note from eqn. (4) that $U_A > 0$ and $U_{AA} < 0$ for all $\phi < 1$ and all β. However, the signs of U_N and U_{NN} are determined by β and ϕ together. To have $U_N > 0$ (so that adding another person to the population increases welfare), we need $(\beta - 1)(1 - \phi) + \phi > 0$. If $\phi \geq 1/2$, this holds for any $\beta > 0$, but if $\phi < 1/2$, then $U_N > 0$ only if $\beta > (1 - 2\phi)/(1 - \phi)$.

Given eqn. (4) for $U(A, N)$, we can modify eqn. (2) to write welfare at time 0 as

$$V_0 = \frac{1}{1-\gamma} \mathcal{E}_0 \int_0^{\infty} [g(A_t, N_t)]^{1-\gamma} e^{-\rho t} dt \ , \tag{5}$$

where

$$g(A_t, N_t) = (\theta A_t)^{1-\phi} N_t^{\omega} \ , \tag{6}$$

[11]Early evidence goes back to Kuznets (1960). More recent studies supporting $\beta > 1$ include Kremer (1993) and Peters (2021). The value of β is likely to change over time and vary with per-capita income; see, e.g., Kelley (1988), Kelley and Schmidt (1994), and Robinson and Srinivasan (1997).

and

$$\omega \equiv \beta(1-\phi) + 2\phi - 1 \ .$$

To complete the model, we need to describe the evolution of A_t and N_t. I assume that both follow independent geometric Brownian motions:

$$dA/A = (r - \theta)dt + \sigma_A dz_A \,, \tag{7}$$

$$dN/N = ndt + \sigma_N dz_N \ . \tag{8}$$

Here, r can be interpreted as the expected return on (broadly defined) capital, and n is the expected rate of population growth. Because consumption is a proportion of wealth, we can write $(r - \theta)$ as the drift of dA/A with no loss of generality.

2.2 Sustainable and Optimal Consumption.

We want the maximum value of θ consistent with the sustainability constraint, i.e., the value $\theta_{\max}$ that just satisfies the constraint that welfare (5) is not expected to decline. Therefore we need to find the drift (expected rate of change) of this integral. For comparison, we also want to find the optimal value of θ, i.e., the value θ_{opt} that maximizes welfare (and may or may not be greater than $\theta_{\max}$).

Given the functional form of $g(A_t, N_t)$ in eqn. (6) and the assumption that A_t and N_t follow GBMs, V_t follows a GBM, so I can apply the approach used by CM to find $\theta_{\max}$ and θ_{opt}. To find $\theta_{\max}$, note that V_t is proportional to $\frac{1}{1-\gamma}[g(A_t, N_t)]^{1-\gamma}$, which is negative because $\gamma > 1$.Thus the sustainability constraint implies that $X_t \equiv [g(A_t, N_t)]^{1-\gamma}$ is not expected to increase, i.e., the drift of dX_t/X_t must be non-positive. To find θ_{opt}, I solve for V_0 and maximize with respect to θ.

Sustainable Consumption-Wealth Ratio.

To find the maximum consumption-wealth ratio consistent with sustainability, we need the drift of dX_t/X_t. Substituting (6) for $g(A_t, N_t)$ yields the following expression for X_t:

$$X_t = (\theta A_t)^{(1-\phi)(1-\gamma)} N_t^{\omega(1-\gamma)} \tag{9}$$

(Recall that $\omega \equiv \beta(1-\phi) + 2\phi - 1$.) Then, using Ito's Lemma,

$$\begin{aligned} dX_t/X_t &= (1-\gamma)\{(1-\phi)(r-\theta) + \omega n + \tfrac{1}{2}(1-\phi)[(1-\phi)(1-\gamma)-1]\sigma_A^2 \\ &+ \tfrac{1}{2}\omega[\omega(1-\gamma)-1]\sigma_N^2\}dt + (1-\gamma)(1-\phi)\sigma_A dz_A + \omega(1-\gamma)\sigma_N dz_N \end{aligned} \tag{10}$$

Since $(1-\gamma) < 0$, the constraint implies that the bracketed part of the drift of dX/X must be non-negative, i.e.,

$$(1-\phi)(r-\theta) + \omega n + \tfrac{1}{2}(1-\phi)[(1-\phi)(1-\gamma)-1]\sigma_A^2 + \tfrac{1}{2}\omega[\omega(1-\gamma)-1]\sigma_N^2 \;\geq\; 0\,. \tag{11}$$

This can be rewritten as:

$$\theta_{\max} = r + \left(\frac{\omega}{1-\phi}\right) n + \frac{1}{2}[(1-\phi)(1-\gamma)-1]\sigma_A^2 + \frac{1}{2}\left(\frac{\omega}{1-\phi}\right)[\omega(1-\gamma)-1]\sigma_N^2 \tag{12}$$

Note that as long as the parameters r, ϕ, n, etc. remain constant, $\theta_{\max}$ will be constant, as originally assumed. Also, $\theta_{\max}$ does not depend on the discount rate ρ, a social preference parameter that values utility received in the future versus today. As shown below, the unconstrained optimal consumption-wealth ratio θ_{opt} does depend on ρ, but $\theta_{\max}$ is a limit that ensures consumption is sustainable over time, irrespective of society's time preference. However, $\theta_{\max}$ does depend on γ, which also reflects social preferences, via σ_A and σ_N. As can be seen from eqn. (9), expected future welfare is a nonlinear function of A and N, and thus is impacted by stochastic fluctuations in A and N. An increase in γ increases the curvature of the welfare function, and thus increases the sizes of those impacts.

Optimal Consumption-Wealth Ratio.

To obtain θ_{opt}, start with dg_t/g_t. From (6), (7), and (8),

$$dg_t/g_t = [(1-\phi)(r-\theta) + \omega n - \tfrac{1}{2}\phi(1-\phi)\sigma_A^2 + \tfrac{1}{2}\omega(\omega-1)\sigma_N^2]dt + (1-\phi)\sigma_A dz_A + \omega\sigma_n dz_N \tag{13}$$

By Ito's Lemma, $d\log g_t = dg_t/g_t - \frac{1}{2}(dg_t/g_t)^2$, so

$$d\log g_t = [(1-\phi)(r-\theta) + \omega n - \tfrac{1}{2}(1-\phi)\sigma_A^2 - \tfrac{1}{2}\omega\sigma_N^2]dt + (1-\phi)\sigma_A dz_A + \omega\sigma_n dz_N\,. \tag{14}$$

Integrate this forward and exponentiate to get g_t:

$$g_t = g_0 \exp\{[(1-\phi)(r-\theta) + \omega n - \tfrac{1}{2}(1-\phi)\sigma_A^2 - \tfrac{1}{2}\omega\sigma_N^2]t + (1-\phi)\sigma_A z_A + \omega\sigma_N z_N\}\,, \tag{15}$$

where $g_0 = (\theta A_0)^{1-\phi} N_0^{\omega}$. Now raise g_t to the power $(1-\gamma)$, substitute into eqn. (5) and integrate to get the following expression for V_0:

$$V_0 = \frac{(\theta A_0)^{(1-\phi)(1-\gamma)} N_0^{\omega(1-\gamma)}}{(1-\gamma)\rho - (1-\gamma)^2[(1-\phi)(r-\theta) + \omega n - \frac{1}{2}(1-\phi)\sigma_A^2 - \frac{1}{2}\omega\sigma_N^2]} \tag{16}$$

Maximizing V_0 with respect to θ yields θ_{opt}:

$$\theta_{\text{opt}} = \frac{\rho + (\gamma-1)[(1-\phi)r + \omega n - \frac{1}{2}(1-\phi)\sigma_A^2 - \frac{1}{2}\omega\sigma_N^2]}{\gamma(1-\phi)+\phi} \tag{17}$$

The sustainability constraint is binding when $\theta_{\max}/\theta_{\text{opt}} < 1$. As one would expect, θ_{opt} is increasing in ρ, because a higher ρ means that society wants more utility today versus in the future, irregardless of the impact on sustainability. Thus the constraint is more likely to be binding if ρ is large. In fact, if $\sigma_N = 0$, the constraint will be binding if $\rho > \theta_{\max}$, as shown by CM. Later we will see how $\theta_{\max}/\theta_{\text{opt}}$ depends on σ_N, r, n, and the other parameters.

2.3 Sustainable Consumption when Lives Have No Intrinsic Value.

I introduced population N into the social welfare function to account for three things: (1) We expect welfare to depend on per capita consumption C/N rather than total consumption C. (2) People are needed to produce, so C is an increasing function of N; in this model $C_t = \theta A_t N_t^{\beta}$, with $\beta > 0$. (3) Society might value the very existence of people apart from their consumption and contribution to production, so that $\phi > 0$ in eqn. (3).

Letting welfare depend on per-capita consumption and letting consumption be an increasing function of population should not be controversial. But letting population affect welfare apart from its contribution to C/N and C might appear strange to some readers. Therefore, it is useful to begin by assuming that lives have no intrinsic social value, i.e., $\phi = 0$. This will provide a link to the existing literature, and also a reference case to which we can compare sustainable consumption when lives do have intrinsic social value.

If $\phi = 0$, $\omega \equiv \beta(1-\phi) + 2\phi - 1 = \beta - 1$, and eqn. (12) for $\theta_{\max}$ becomes:

$$\theta_{\max} = r + (\beta-1)n - \tfrac{1}{2}\gamma\sigma_A^2 - \tfrac{1}{2}(\beta-1)[(\beta-1)(\gamma-1)+1]\sigma_N^2 \tag{18}$$

With no uncertainty, $\theta_{\max} = r + (\beta-1)n$. If $\beta = 1$, population growth does not affect per-capital consumption, so we have the standard sustainability constraint $\theta_{\max} = r$, i.e., the requirement that $r - \theta \geq 0$, which ensures that consumption does not draw down total productive wealth.[12] If $\beta < (>)\ 1$, then $\theta_{\max} = r + (\beta - 1)n < (>)\ r$, i.e., a lower (higher) consumption-wealth ratio will ensure that total productive wealth does not decline.

Now suppose population growth is certain ($\sigma_N = 0$) but the return on capital is uncertain ($\sigma_A > 0$). Then $\theta_{\max} = r + (\beta - 1)n - \frac{1}{2}\gamma\sigma_A^2$, so σ_A^2 reduces $\theta_{\max}$, relative to the expected return on capital, r.[13] As we will see, this reduction in $\theta_{\max}$ can be substantial.

The dependence of $\theta_{\max}$ on $\sigma_N > 0$ is more complex. First, uncertainty over population growth matters only if $\beta \neq 1$. If $\beta = 1$, an increase in N results in a commensurate increase in C, leaving welfare unchanged. If $\beta \neq 1$, an increase in N alters per-capita consumption. But note from eqn. (12) that $\theta_{\max}$ is also independent of σ_N if $\beta = (\gamma - 2)/(\gamma - 1)$. In general, the sign of $\partial\theta_{\max}/\partial\sigma_N^2$ depends on β and γ. With $\phi = 0$,

$$\frac{\partial\theta_{\max}}{\partial\sigma_N^2} = \begin{cases} > 0 & \text{if} \quad (\gamma-2)/(\gamma-1) < \beta < 1 \\ 0 & \text{if} \quad \beta = (\gamma-2)/(\gamma-1) \quad \text{or} \quad \beta = 1 \\ < 0 & \text{if} \quad \beta < (\gamma-2)/(\gamma-1) \quad \text{or} \quad \beta > 1 \end{cases}$$

This dependence of $\partial\theta_{\max}/\partial\sigma_N^2$ on β and γ is illustrated in Figure 1, and follows from the fact that in eqns. (5) and (6) for welfare, when $\phi = 0$ the exponent on N is $(\beta - 1)(1 - \gamma)$, which is $< (>)\ 0$ if $\beta > (<)\ (\gamma - 2)/(\gamma - 1)$, so that stochastic fluctuations in N_t increase (reduce) welfare and thus $\theta_{\max}$.

When lives have no intrinsic value and there is no uncertainty, $\theta_{\max}$ depends only on the expected return on capital r, the expected population growth rate n, and the elasticity of consumption with respect to population β. Apart from this last term, it follows the earlier literature, and the parameter γ plays no role. When the return on capital is stochastic ($\sigma_A > 0$), $\theta_{\max}$ is reduced by $\frac{1}{2}\gamma\sigma_A^2$. A reasonable value for σ_A is 0.20, which is roughly the annual standard deviation of returns on the S&P500. Even if $\beta = 1$, a value of γ above 2 can drive $\theta_{\max}$ below zero, so that there is *no* sustainable level of consumption. As for stochastic fluctuations in population growth, we saw that they can raise or lower $\theta_{\max}$, depending on β and γ, but the impact is small. Historical values of σ_N vary across countries, but are usually around 0.01 or .02. Then if $\gamma = 4$ and $\beta = 0$, $\theta_{\max}$ would fall by .0004 or less.

[12] This is equivalent to the constraint in Dasgupta (2009) and Arrow et al. (2012) that there is no reduction in "comprehensive wealth." If $\beta = 1$, the sustainability constraint is binding (i.e., $\theta_{\max} < \theta_{\text{opt}}$) if $\rho > r$. If $\beta = 0$, the constraint is binding if $\rho > (r - n)$.

[13] This differs from CM, who find σ_A^2 increases $\theta_{\max}$, but relative to the risk-free rate. In CM $r = r_f + \mu$, where r_f is the risk-free rate, μ is the expected excess return, and $\mu = \gamma\sigma_A^2$, so $r = r_f + \gamma\sigma_A^2$. In my model r is constant and independent of σ_A.

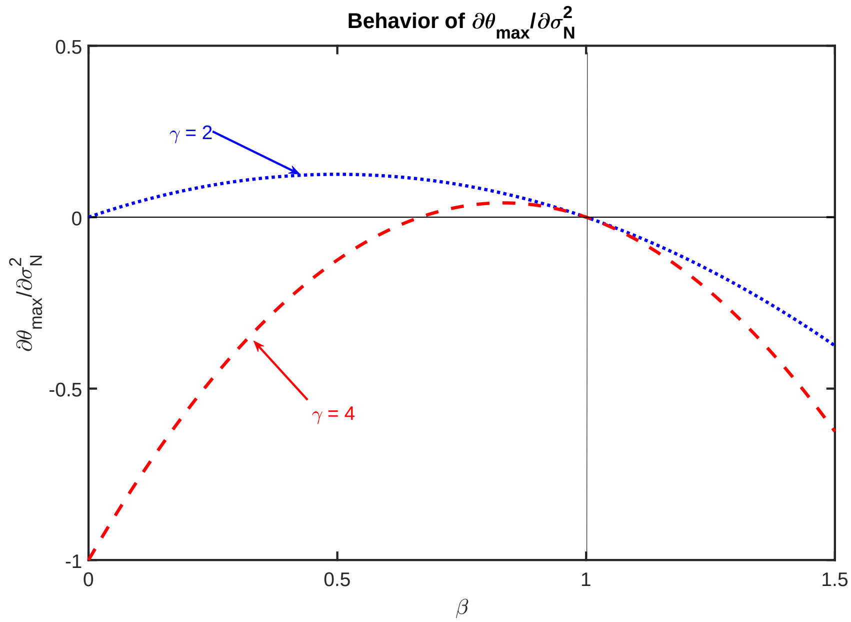

Figure 1: Dependence of $\partial\theta_{\max}/\partial\sigma_N^2$ on β and γ when $\phi = 0$. If $\gamma = 2$, $\partial\theta_{\max}/\partial\sigma_N^2 \geq 0$ for all values of β between 0 and 1. Increasing γ increases the range over which $\partial\theta_{\max}/\partial\sigma_N^2 < 0$.

In the next section I explore how $\theta_{\max}$ changes when lives have intrinsic social value. I begin by ignoring uncertainty, and then later allow σ_A and σ_N to be positive.

3 Sustainability in a Deterministic World.

In most models, population growth reduces sustainable consumption, which must be spread among more people. An exception is when an increase in population proportionally or more than proportionally increases total consumption ($\beta \geq 1$ in my model). But when lives have intrinsic social value, population growth can affect sustainable consumption via a different route. With $\sigma_A = \sigma_N = 0$, we will see how a preference for more or fewer people affects $\theta_{\max}$.

3.1 Lives versus Consumption.

If β is sufficiently large and $\phi > (<)\ 0$, $\theta_{\max}$ increases (decreases) when n increases, because population growth and consumption growth become substitutes in terms of their contributions to welfare. With $\phi > 0$, the welfare gain from a growing population can partially offset the loss from reduced future consumption, so productive wealth can be drawn down faster by increasing current consumption (raising θ). Note that $g(A_t, N_t)$ in eqns. (5) and (6) is

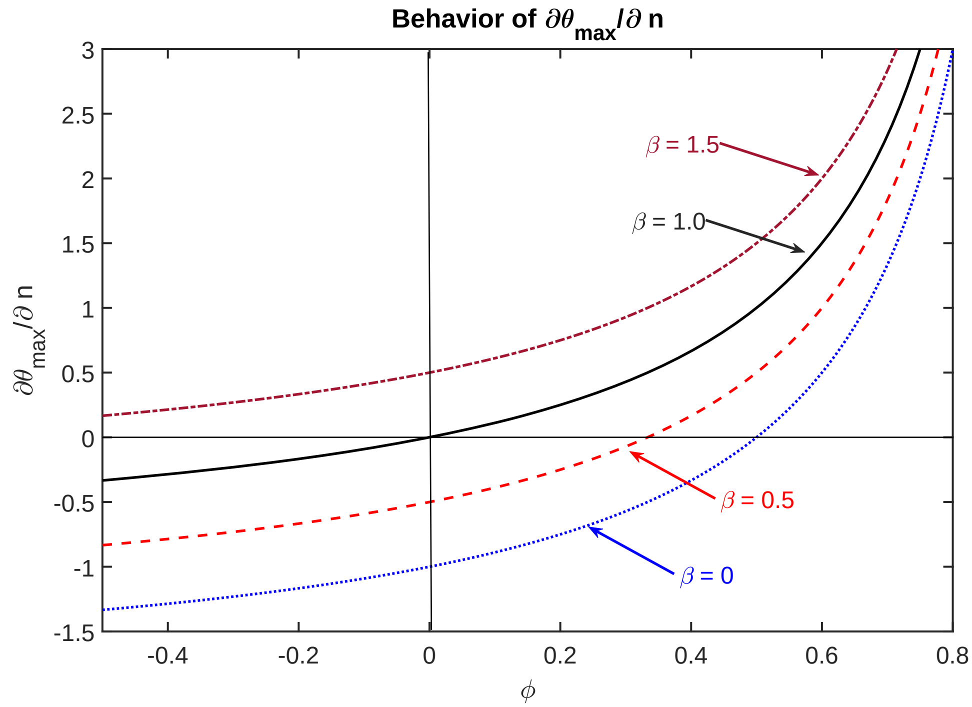

Figure 2: Dependence of $\partial\theta_{\max}/\partial n$ on ϕ for different values of β. An increase in β implies an increase in the contribution of population to total consumption, raising $\partial\theta_{\max}/\partial n$. For any value of β, $\partial\theta_{\max}/\partial n$ is increasing in ϕ. If $\phi > 0$, growth in N_t adds to welfare, raising $\theta_{\max}$ so wealth can be drawn down faster. This reduces future consumption and thus welfare, but is sustainable because it is offset by the increase in welfare from a larger future population.

just $g(A_t, N_t) = (\theta A_t N_t^{\beta-1})^{1-\phi} N_t^{\phi} = (C_t/N_t)^{1-\phi} N_t^{\phi}$. Thus changes in expected welfare can result from changes in N_t (if $\phi \neq 0$) and/or changes in (C_t/N_t). Holding N_t fixed, future consumption can increase via growth in A_t or via growth in productive wealth from reducing current consumption (i.e., reducing θ).

Figure 2 shows $\partial\theta_{\max}/\partial n$, i.e., how $\theta_{\max}$ changes in response to a change in the expected rate of population growth n, for several values of β. Start with $\beta = 1$ (the solid black curve), so that population growth leaves per-capita consumption unchanged. Then if $\phi = 0$ so only C_t/N_t affects welfare, a change in n has no effect on $\theta_{\max}$. But if $\phi > 0$, there is a welfare gain from faster population growth, allowing current consumption to be sustainably increased, so $\partial\theta_{\max}/\partial n > 0$. The increase in $\theta_{\max}$ implies that wealth will be drawn down, reducing future consumption, but the resulting welfare loss is offset by the gain from a growing population.

As Figure 2 shows, this effect holds for any value of β. If $\beta = 0$ so that growth in N_t does not increase consumption, $\partial\theta_{\max}/\partial n = -1$ when $\phi = 0$ (as in standard models of sustainable consumption), but increases with ϕ, and is positive when $\phi > \frac{1}{2}$. Again, a higher value of

$\theta_{\max}$ means wealth is drawn down faster, but this is sustainable because the welfare loss from reduced future consumption is offset by the gain from a larger population.

What if $\phi < 0$, so that a growing population has a negative social value? Again, suppose $\beta = 1$ so population growth leaves C_t/N_t unchanged. Now an increase in n reduces $\theta_{\max}$, because the welfare loss from faster population growth must be offset by a welfare gain from greater future consumption. But faster consumption growth requires growth in productive wealth, and thus a reduction in current consumption, so $\theta_{\max}$ is lower.

3.2 Bounds on Population Growth.

Figure 2 shows how a change in the mean rate of population growth n affects the maximum sustainable consumption-wealth ratio $\theta_{\max}$, reducing it if $\phi \leq 0$ and $\beta < 1$, and possibly increasing it if $\phi > 0$ or $\beta > 1$. But depending on ϕ and β, a sufficiently high—or low—value of n could result in $\theta_{\max} < 0$, so that there is *no* sustainable level of consumption. I define the *critical* values of n as the upper and lower bounds at which $\theta_{\max} = 0$.

This is illustrated in Figure 3, which shows the critical population growth rates n_c as a function of ϕ for $\beta = 0$ (blue dotted line), $\beta = 0.5$ (red dashed line), and $\beta = 1.0$ (green dot-dash line). The vertical lines are at values of ϕ for which $\theta_{\max}$ is independent of n.

Suppose $\beta = 1$, so $\theta_{\max} = r + \phi n/(1 - \phi)$ is independent of n when $\phi = 0$. If $\phi < 0$ so lives have negative social value, any population growth will reduce welfare, and if population growth is large enough, it can drive $\theta_{\max}$ below zero. In Figure 3, $\theta_{\max} > 0$ only if $n < n_c = -r(1 - \phi)/\phi$, i.e., n is below the green dashed line at the top left corner of the figure. If $n > n_c$, then even if we reduce current consumption to zero in order to build up wealth and thereby increase future consumption, the resulting welfare gain will be smaller than the loss from a growing population. But if $\phi > 0$, the constraint is turned around. Now $n_c < 0$, and we need $n > n_c$ to have $\theta_{\max} > 0$. If $n < n_c$, the welfare loss from a falling population cannot be offset by the gain from increasing future consumption, even by consuming nothing today.

Suppose $\beta = 0$, so a doubling of population cuts C_t/N_t consumption in half. Now $\theta_{\max} = r + (2\phi - 1)n/(1 - \phi)$, which is independent of n if $\phi = 0.5$, and $n_c = r(1 - \phi)/(1 - 2\phi)$, shown as the blue dotted line in Figure 3. If $\phi = 0$, sustainable consumption requires $n < n_c = r$. If $\phi = 0.2$, $n_c = 1.33r = .08$ with $r = .06$. So if $n = .09$ as at point A in the figure, then $\theta_{\max} = -.01$, and no level of consumption is sustainable. But if n is only .02, then $\theta_{\max} = .045$. If society consumes at this level, wealth is drawn down, but the welfare loss from reduced consumption growth is offset by the gain from population growth. For

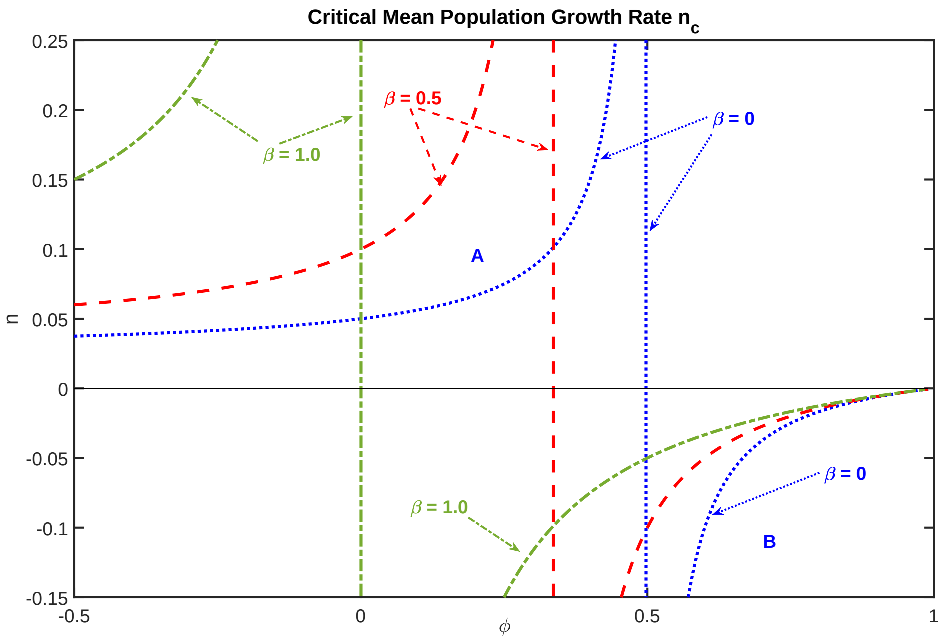

Figure 3: Critical population growth rates, n_c, at which $\theta_{\max} = 0$. Here $r = .06$ and $\sigma_A = \sigma_N = 0$. If $\beta = 1$ (green dot-dash line), $\theta_{\max} = r + \phi n/(1-\phi)$ is independent of n when $\phi = 0$. If $\phi \neq 0$, n must be between the green lines to have $\theta_{\max} \geq 0$. If $\beta = 0$ (blue dotted line), $\theta_{\max} = r + (2\phi - 1)n/(1-\phi)$, so if $\phi = .2$ and $n = .09$ as at point A, $\theta_{\max} = -.01$, and no level of consumption is sustainable. For large ϕ, n must not be too low. If $\phi = .7$, $n_c = -.045$, so if $n = -.10$ (point B), $\theta_{\max} = -.073$; now no level of consumption yields enough welfare to offset the loss from a rapidly declining population.

large ϕ, population growth must not be too low. If $\phi = .7$, $n_c = -.045$, so if $n = -.10$ (as at point B), $\theta_{\max} = -.073$, so no level of consumption provides enough welfare to offset the loss from a rapidly declining population. Welfare is unsustainable, not because of too much consumption but because of too few people.

As Figure 3 shows, depending on the value society places on lives, population growth can be too high—or too low—to allow for any sustainable level of consumption.

4 Population Growth and Sustainability.

Population growth affects both the sustainable and optimal consumption-wealth ratios, but in different ways. To see this we show how $\theta_{\max}/\theta_{\text{opt}}$ varies with n. Can changing n drive this ratio below 1, and how does the answer depend on the extent to which lives have intrinsic social value? We begin with no uncertainty, and then allow σ_A and σ_N to be positive.

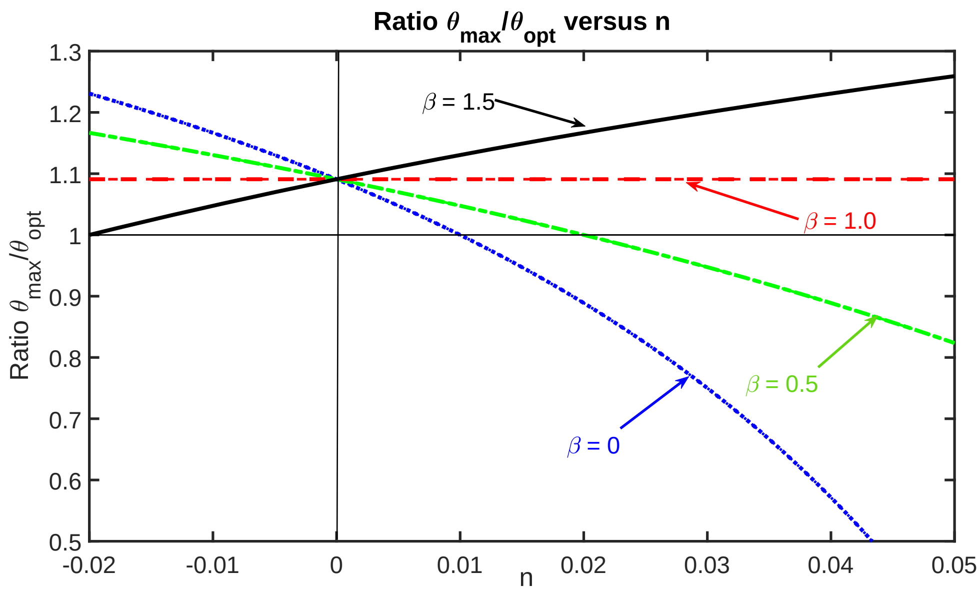

Figure 4: The ratio $\theta_{\max}/\theta_{\text{opt}}$ versus the mean population growth rate n, for $r = .06$, $\rho = .05$, $\gamma = 2$, $\sigma_A = \sigma_N = 0$, and $\phi = 0$. If $\beta = 0$ or 0.5, the ratio declines as n increases because population growth contributes little or nothing to output and has no intrinsic value ($\phi = 0$). If $\beta = 1.5$, population growth raises per-capita consumption, so the ratio increases with n.

4.1 No Uncertainty.

Figure 4 shows how the ratio $\theta_{\max}/\theta_{\text{opt}}$ varies with n when $\phi = 0$, and $r = .06$, $\rho = .05$, $\gamma = 2$, and $\sigma_A = \sigma_N = 0$. If $n = 0$, i.e., no population growth, the ratio is independent of β. If $\beta = 0$ (the blue dotted line in the figure), population growth is a pure burden on welfare because it contributes nothing to output and has no intrinsic value ($\phi = 0$), so the ratio declines with n, falling below 1 at $n = .01$. The decline is less steep if $\beta = 0.5$, and if $\beta = 1$, population growth leaves C_t/N_t unchanged, so the ratio is independent of n. Finally, if $\beta = 1.5$, population growth raises per-capita consumption, so the ratio increases with n.

In this model the relationship between population growth and total consumption is symmetric; if $\beta = 1.5$ ($\beta = 0$) negative population growth will reduce (increase) per-capita consumption. So when $n < 0$, $\theta_{\max}/\theta_{\text{opt}}$ is larger for $\beta = 0$ than for $\beta > 0$.

Figure 5 also shows $\theta_{\max}/\theta_{\text{opt}}$ versus n, but now with $\phi = 0.3$, so population growth has positive social value. Now population growth affects welfare through its intrinsic value and (as before) through its effect on per-capita consumption. When $\beta = 0.5$ the two effects nearly offset each other; a higher n reduces welfare by reducing per-capita consumption but increases welfare via the intrinsic value of more people, leaving the ratio almost unchanged

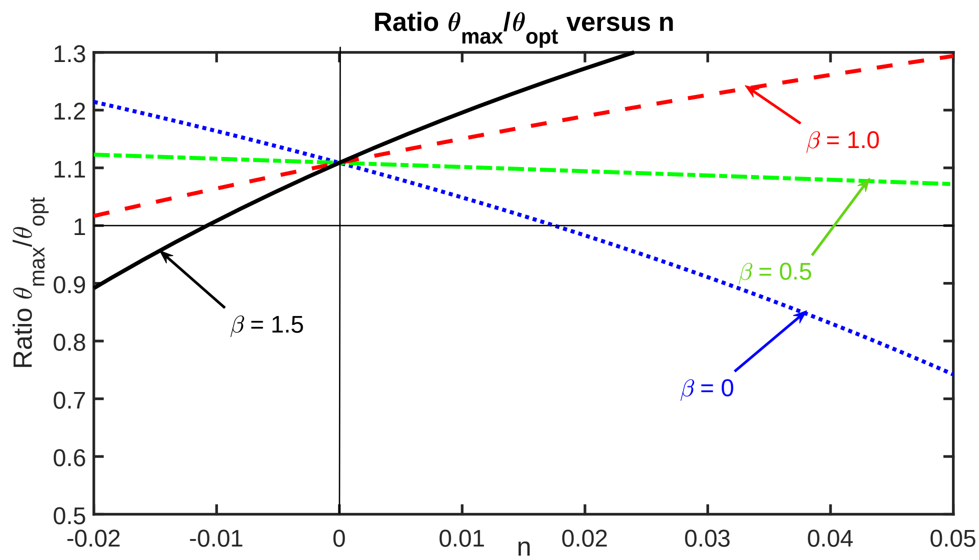

Figure 5: The ratio $\theta_{\max}/\theta_{\text{opt}}$ versus the mean population growth rate n, for $r = .06$, $\rho = .05$, $\gamma = 2$, $\sigma_A = \sigma_N = 0$, and $\phi = 0.3$, so population growth has positive social value. Now if $\beta = 1$ the ratio increases with n (and increases rapidly if $\beta = 1.5$).

as n increases. If $\beta = 0$, the ratio declines with n, but less rapidly because a larger population increases welfare. If $\beta = 1$ so that population growth leaves per-capita consumption unchanged, the ratio increases with n, and increases rapidly if $\beta = 1.5$. If $\beta = 1.5$ a *negative* value of n reduces social welfare in two ways, by reducing per-capita consumption and by reducing the intrinsic value of the (shrinking) population.

4.2 Risk and the Sustainability Constraint.

We now introduce uncertainty over the evolution of A_t and N_t. Stochastic fluctuations in A_t reduce $\theta_{\max}$, and depending on β and γ, fluctuations in N_t can also affect $\theta_{\max}$. A reasonable value for σ_A is 0.2, and we will see that this can significantly reduce $\theta_{\max}/\theta_{\text{opt}}$ for every value of ϕ. Depending on β, if $\sigma_A = 0.2$, $\theta_{\max}/\theta_{\text{opt}}$ can be well below 1, unless ϕ is large.

Figure 6 shows the ratio $\theta_{\max}/\theta_{\text{opt}}$ as a function of ϕ, for $n = .02$, $r = .06$, $\rho = .05$, $\gamma = 2$, $\sigma_N = 0.02$, $\beta = 0$ and 1, and $\sigma_A = = 0$ and 0.2. Start with $\sigma_A = 0$, shown by the two lines starting close to $\theta_{\max}/\theta_{\text{opt}} = 1$. If $\beta = 0$ (the green dotted line), per-capita consumption is falling (because $n = .02$), so $\theta_{\max}/\theta_{\text{opt}} < 1$ when $\phi = 0$. But the ratio increases as ϕ increases and the gain in social value from a growing population eventually outweighs the loss from the lower per-capita consumption. If $\beta = 1$ (the solid black line), population growth leaves per-capita consumption unchanged, and $\theta_{\max}/\theta_{\text{opt}}$ is always above 1.

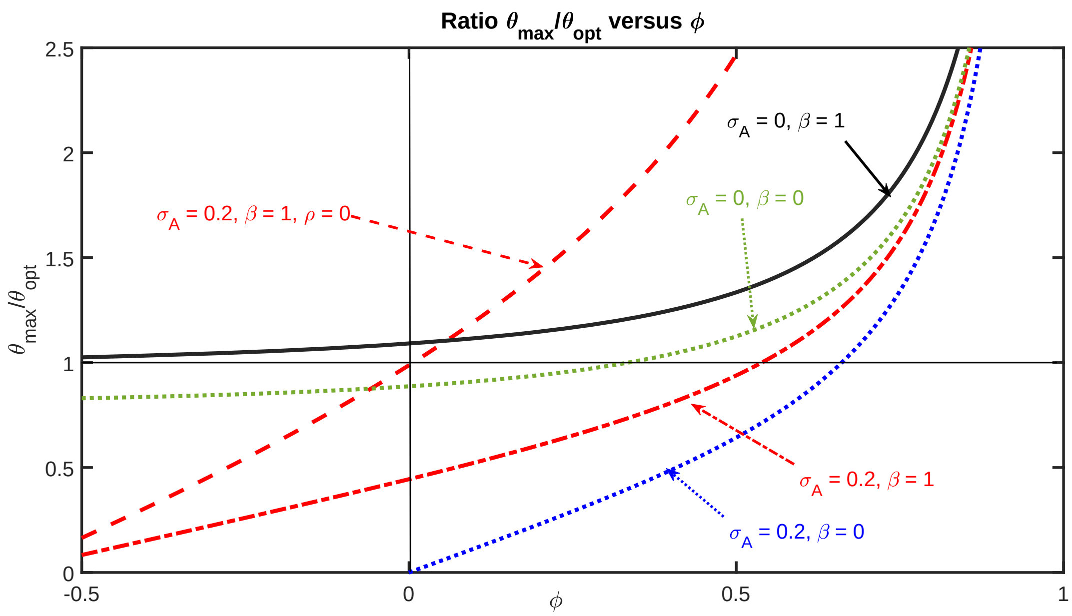

Figure 6: The Ratio $\theta_{\max}/\theta_{\text{opt}}$ versus ϕ for $n = .02$, $r = .06$, $\rho = .05$, $\gamma = 2$, $\sigma_N = 0.02$, $\beta = 0$ and 1, and $\sigma_A = = 0$ and 0.2. For the two lines starting close to $\theta_{\max}/\theta_{\text{opt}} = 1$, $\sigma_A = 0$. If $\beta = 0$ (green dotted line), C_t/N_t is falling (because $n = .02$), so $\theta_{\max}/\theta_{\text{opt}} < 1$ if $\phi = 0$. But the ratio increases with ϕ because of the gain in social value from a growing population. If $\beta = 1$ (solid black line), population growth leaves C_t/N_t unchanged, and $\theta_{\max}/\theta_{\text{opt}} > 1$ always. When $\sigma_A = 0.2$, both curves are lower, and the ratio is well below 1 unless ϕ is large. (If $\beta = 0$, as in the blue dotted line, there is no sustainable consumption if $\phi \leq 0$.) But the ratio rises sharply if $\rho = 0$ (the red dashed line for $\beta = 1$), and is above 1 if $\phi > 0$.

For the two curves at the bottom of the figure, $\sigma_A = 0.2$. Now unless ϕ is very large, $\theta_{\max}/\theta_{\text{opt}}$ is below 1 for both $\beta = 0$ and 1. If $\beta = 0$ (blue dotted line), $\theta_{\max}/\theta_{\text{opt}} < 0$ if $\phi < 0$, so that no positive level of consumption is sustainable. If $\beta = 1$ (red dash-dot line), $\theta_{\max}/\theta_{\text{opt}} \approx 0.4$ if $\phi = 0$, and is above 1 only if $\phi > 0.5$ Also, in this figure, $\gamma = 2$; a higher value of γ will push the ratio down.

The two curves at the bottom of Figure 6 ($\sigma_A = 0.2$) imply that the sustainable consumption-wealth ratio will be well below the optimal ratio that maximizes social welfare. That means sustainability will impose a welfare cost on society, the size of which we will examine in the next section. However, there is one parameter that we can consider in more detail, namely the discount rate ρ, i.e., the rate of time preference. I set $\rho = .05$ because that is consistent with numbers in the macroeconomics and finance literatures, and with experimental evidence on people's time preferences. But one could argue that the value of ρ need not reflect preferences, but instead is a normative number that should be based on how society trades off the welfare of future versus current generations. As in the debate over the "correct" discount rate for climate change policy, one could argue that ρ should be much lower than a "market" value, perhaps .01 or .02, or (as Ramsey argued) even 0.

Reducing ρ to zero will substantially reduce the optimal consumption-wealth ratio, and thereby raise $\theta_{\text{max}}/\theta_{\text{opt}}$, as shown by the red dashed line in Figure 6, for which $\sigma_A = 0.2$ and $\beta = 1$, but $\rho = 0$ instead of .05. With this change, $\theta_{\text{max}}/\theta_{\text{opt}}$ is well above 1 for all values of $\phi > 0$. This suggests that support for government policies focused on sustainability may have to rely on "ethical" arguments for parameters such as ρ. In this sense the discount rate can play a critical role in "sustainability policy" just as it does in climate change policy.

5 The Social Cost of Sustainability.

We have seen that for plausible parameter values, the sustainable consumption-wealth ratio is well below the optimal ratio, which implies that sustainability would impose a welfare cost on society. How large is that cost? To find out, we use eqn. (5) to compare welfare at time $t = 0$ when $\theta = \theta_{\text{opt}}$ (denoted by V_{opt}) to welfare when $\theta = \theta_{\text{max}}$ (denoted by V_{max}). Because $\gamma > 1$, V_{opt} and V_{max} are both negative, so this comparison can be expressed in terms of the following percentage welfare loss:

$$\text{Loss} = (V_{\text{max}} - V_{\text{opt}})/V_{\text{max}} \, . \tag{19}$$

This percentage loss is shown in Figure 7 for three values of ρ, and for $n = .02$, $r = .06$, $\beta = 1$, $\gamma = 2$, $\sigma_N = 0.02$, and $\sigma_A = 0.2$. For the "base case" calculation, $\rho = .05$ and the welfare loss is considerable unless ϕ is large; if $\phi = 0$, the loss is over 30%, and it becomes zero only if $\phi > 0.4$. Doubling ρ to 0.10 makes the loss much larger; about 55% if $\phi = 0$. Also, for any ρ, the percentage loss is large for negative values of ϕ. The reason is that if a growing population has negative social value (as some environmentalists would argue it does), sustainability requires it to be offset by the positive value of greater consumption growth. But greater consumption growth requires a reduction in current consumption so that productive wealth can increase.

Even if we restrict the analysis to $\phi \geq 0$, the welfare loss can be substantial if ρ is .05 or greater. But there is no loss if $\rho = 0$. (Reducing ρ reduces θ_{opt} relative to θ_{max}.) This brings us back to the question of whether ρ should reflect people's actual time preferences, or instead should be treated as a normative number based on ethical views of how society

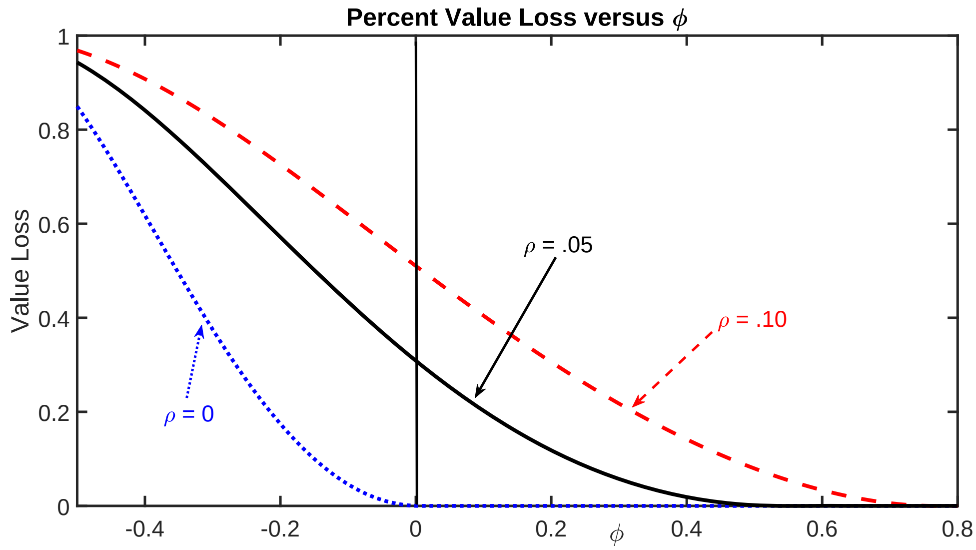

Figure 7: Percentage Loss of Value from Sustainability Constraint. Loss is given by eqn. (19).

should trade off the welfare of future versus current generations. This is another way of stating the more general question of whether sustainability, as opposed to the maximization of social welfare, should be the objective of government policy.

6 Conclusions.

Much of the literature on sustainability focuses on consumption, and defines a sustainable consumption path as one for which social welfare never declines. But that literature is largely deterministic and treats population growth as incidental. In this paper social welfare depends on consumption and population, both of which evolve stochastically. In addition, population can affect social welfare through its impact on aggregate output (and hence consumption), but also through its intrinsic social value.

The model in this paper is simple, with a small set of parameters. But determining the values of those parameters is not straightforward, and raises questions about sustainability as a social objective. Notably, a high value of the discount rate ρ means society wants utility, and hence consumption, now rather than later, possibly in conflict with the sustainability constraint. Should society be bound by that constraint and reduce its current consumption to benefit future generations, rather than consuming at the higher level that maximizes social welfare? Can we argue that ρ should be set close to zero on "ethical" grounds, which could put the sustainable level of consumption above the optimal level? This dilemma is analogous to the discount rate problem in climate change policy: A high market-based discount rate will imply an optimal policy of limited CO_2 emission abatement, at a cost to future generations, leading some to argue that a very low ethics-based discount rate should be used instead.

Another difficult problem is how to determine a value for ϕ. How should we decide whether human lives have intrinsic value, and what that value is relative to the value of consumption? We have shown that sustainable consumption can depend critically on the (positive or negative) value that society places on lives. There is probably no "correct" value for ϕ; instead this parameter (and the underlying model) should be viewed as a vehicle for exploring how an intrinsic value of lives can affect sustainable consumption.

More generally, plausible parameter values put the sustainable consumption-wealth ratio below the optimal ratio that maximizes welfare. This result is reversed if society places a large positive value on lives, the elasticity of output with respect to population is close to or above one, and the volatility of the return on capital is low. But without these conditions, the goal of sustainability creates a policy dilemma. Should we reduce consumption to a sustainable level, even if this pushes social welfare below what it could be otherwise? There are some who would argue that sustainability is more important than maximizing welfare. But in that case we should be aware of the costs of sustainability, which we have seen can be substantial.

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