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Consumer Subsidies with a Strategic Supplier: Commitment vs. Flexibility

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Governments use consumer incentives to promote green technologies (e.g., solar panels and electric vehicles). Our goal in this paper is to study how policy adjustments over time will interact with production decisions from the industry. We model the interaction between a government and an industry player in a two-period game setting under uncertain demand. We show how the timing of decisions affects the risk-sharing between government and supplier, ultimately affecting the cost of the subsidy program. In particular, we show that when the government commits to a fixed policy, it encourages the supplier to produce more at the beginning of the horizon. Consequently, a flexible subsidy policy is on average more expensive, unless there is a significant negative demand correlation across time periods. However, we show that the variance of the total sales is lower in the flexible setting, implying that the government's additional spending reduces adoption level uncertainty. In addition, we show that for flexible policies, the supplier is better-off in terms of expected profits whereas the consumers can either benefit or not depending on the price elasticity of demand. Finally, we test our insights with a numerical example calibrated on data from a solar subsidy program.

Key words: Government subsidies, Strategic supplier, Newsvendor

1. Introduction

In order to stimulate the adoption of a new technology, governments have typically introduced policy interventions to subsidize customers. Examples of such subsidy programs in Europe and in the US are common in the renewable energy sector, where feed-in-tariffs and rebates have helped promote solar and wind technologies. In Germany, solar electricity contributed to roughly 4.6% of total electricity consumption in 2012. Combined with wind power expansion, the country is well on its way to reach the long term goal of 35% renewable energy by 2020. Dating back to 2001, the German feed-in-tariff program initially paid solar panel owners 50.62 Euro cents per kWh of electricity produced, more than 3 times the average retail electricity price. This feed-in-tariff system kick-started a new solar industry and by the end of 2011 there were more than 24.7 GW of installed photovoltaic capacity in Germany, which represents roughly 37% of the total installed capacity worldwide. Over the years of 2010, 2011, and 2012, Germany has added a consistent 7.4 to 7.6 GW per year of photovoltaic capacity. Achieving this target has not been an easy feat. In 2012, the German feed-in-tariff level has changed 4 times throughout the year[1]. The effect that these policy adjustments have caused on the solar industry is not yet clear and this research aims to shed a new light into this problem.

The federal tax credit for plug-in electric vehicles is another example of subsidy program. Introduced in 2009, the US government provides a consumer subsidy of $7,500 for the purchase of an electric vehicle[2]. Unlike the solar subsidy mentioned before, this subsidy amount has not changed since it's inception. We hope to understand what are the benefits and disadvantages of such policy commitment. Additionally, we hope to understand the implications of the timing of government decisions for the industry and for consumers.

The value of commitment in public policies has been studied in other contexts. As summarized by Dixit and Pindyck (1994): "if an objective of public policy is to stimulate investment, the stability of interest rates may be more important than the level of interest rates". This insight is derived by Ingersoll and Ross (1992), who show that the interest rate uncertainty delays investments. On the other hand, one can find situations where uncertainty is not as harmful to investments. For example, Kulatilaka and Perotti (1998) show that interest rate volatility actually increases the incentive for early investment under a competitive environment. In this paper, we explore this question of policy commitment in the area of subsidy policies. In particular, we measure the trade-off between commitment and flexibility with respect to the production incentives of the suppliers.

In fact, we show how policy revisions interact with a strategic supplier in this market. The anticipation of a policy change decreases the supplier's production target and may increase the overall cost of the subsidy program. Should the government commit their subsidy levels for a longer period of time or should the subsidy policy adapt to the realized market demand after each period?

To study these questions, we model the system as a two-period game between the government and the supplier. The government chooses the subsidy levels for each period and then the supplier chooses it's production levels. Demand is uncertain, so the supplier solves a multi-period newsvendor problem. We compare two game settings: the government commits to a fixed subsidy policy for each period in advance; or the government has a flexible policy that adapts after the first period demand is realized.

[1] International Energy Agency - Photovoltaic Power Systems Programme - Annual Report, 2012

[2] Internal Revenue Service - Notice 2009-89: New Qualified Plug-in Electric Drive Motor Vehicle Credit.

1.1. Contributions

Under a flexible setting, by holding the option of adjusting the subsidy, the government decreases the underage risk of the supplier. In effect, this lowers the supplier's initial production level. Because of that, the subsidy levels are on average higher without policy commitment. This effect grows with the magnitude of demand uncertainty, which presents a counter-intuitive insight. Instead of a hedging effect, the government spending is more exposed to the variance of the demand uncertainty when using a flexible policy.

As a consequence, when looking at total average spending, we observe that under a flexible setting the government typically has to pay a higher cost for achieving the same target level. This difference becomes even larger as demand volatility increases or if profit margins are high in the second period relative to the first. The average flexible spending only becomes lower when there is a strong negative correlation between demand in the two periods. On the other hand, the premium paid for adaptability in the flexible setting provides a lower variance in sales.

Firms will always obtain higher profits with a flexible government policy. Consumers might prefer the flexible or the committed setting depending on the price sensitivity of demand. In particular, if price sensitivity in the second period is much higher than in the first period, the benefits of higher subsidies in the flexible case are outweighed by the probability of undersupplying high-valued customers.

1.2. Literature review

There is a growing literature in operations management that studies the impact of subsidy programs. Some develop a prescriptive model for policy optimization, as for example Lobel and Perakis (2011). Alizamir et al. (2012) show that subsidies should not be designed to keep investor profitability constant. Krass et al. (2013) explore the use of environmental taxes to stimulate adoption of green technologies and argue that subsidies should be used to complement the taxes and reduce the welfare loss. Similarly, Terwiesch and Xu (2012) also show that subsidies are often better to stimulate innovation in green technologies than taxes for the polluting technology. Mamani et al. (2012) and Chick et al. (2014) study how to coordinate a vaccines market with subsidies and how to mitigate information asymmetry. It is important to note that the papers above do not explicitly consider demand uncertainty and the resulting mismatch between demand and supply. Kok et al. (2014) model the supply uncertainty from different renewable generation technologies and show how subsidy policies can obtain different outcomes depending on this uncertainty. On the other hand, demand uncertainty can be a significant issue when promoting a green technology product. For example, Sallee (2011) shows that there was a shortage of vehicles manufactured to meet demand when the Toyota Prius was launched. Ho et al. (2002) also show that because of diffusion effects, the firm might want to delay the product launch to build-up inventory and avoid a later stock-out. This provides further motivation for studying the supplier with a newsvendor model.

Modeling demand uncertainty, Taylor and Xiao (2014b) develop a model for how donors should fund malaria drugs through private retailers. They show that donor funding should subsidize purchases not sales of drugs. Ovchinnikov and Raz (2013) compare subsidizing the manufacturer cost and/or consumer purchases in the presence of a single period price-setting newsvendor. They show that only a joint mechanism can completely coordinate the supply-chain, but using only a consumer rebate typically has a small welfare loss. Taylor and Xiao (2014a) compares subsidizing commercial and non-commercial channels. They show the optimal level of subsidy has a non-trivial relationship with the level of consumer awareness for the product.

Perhaps closer to this paper, Cohen et al. (2013) study the direct impact of demand uncertainty in a single period game setting between the government and the supplier. They model the supplier as a price-setting newsvendor and show that risk is shared between the supplier and the government depending on how profitable the product is. In contrast, this paper explores a two period setting and the impact of game dynamics in the risk-sharing between the government and supplier.

Kaya and Özer (2012) provide a good survey of the literature on inventory risk sharing in a supply chain with a newsvendor retailer. Lutze and Özer (2008) show how demand information and inventory risk can be optimally shared in a supply-chain with lead-times. Babich (2010) shows how a manufacturer can use ordering and subsidy decisions to mitigate the disruption risk from a risky supplier.

The tradeoff between commitment and flexibility has been studied in other applications within the operations management literature. In a supply chain context, Erhun et al. (2008) show that the supplier, buyer and consumers benefit from a multi-stage dynamic procurement, rather than a single wholesale price contract. Granot and Yin (2007) study how a sequential commitment with buy-back contracts can increase supplier's profit, but harm the retailer. When introducing a new product, Liu and Özer (2010) show that sharing updated demand information to the upstream supplier can provide channel benefits, but a quantity flexibility contract is less robust than a buy-back contract. Kim and Netessine (2013) show that commitment to profit margins can be valuable. It fosters collaboration between supplier and manufacturer, while simple commitments to price or quantity do not. Olsen and Parker (2014) show that inventory commitment can be valuable in a dynamic competition between suppliers.

When considering price flexibility in the presence of strategic consumers, the value of commitment tends to dominate the advantages of flexibility. Aviv and Pazgal (2008) show that the retailer has an incentive to commit to a fixed pricing strategy over a flexible strategy. While most of this literature shows that a firm should avoid discounting to prevent strategic customer behavior, Elmaghraby et al. (2008) show that a pre-committed markdown dominates a single fixed price. Cachon and Feldman (2013) also show that when customers incur search costs, the firm should commit to frequent discounts. Volume flexibility can also be a useful tool to mitigate adverse consumer behavior (see Cachon and Swinney (2009)). Yin et al. (2009) show that hiding inventory information from the customers could mitigate some of the customers' strategic response. Lobel et al. (2013) show that committing to a set schedule of product launches is better than having the flexibility to release products over time.

Chod and Rudi (2005) and Chod et al. (2010) argue that flexibility (in pricing or production capacity) is especially important as an instrument to protect the firm against demand variability and correlation. Goyal and Netessine (2011) also show that the value of flexible production capacity depends on the level of demand correlation across different products. In the context of supply chains, Barnes-Schuster et al. (2002) show how flexible contracts with options can further coordinate the supply chain. Anand et al. (2008) show that a dynamic contract is preferred over a committed contract by the supplier, the buyer and consumers. In this case, the flexible contract empowers the buyer and reduces double marginalization, bringing the system to a higher level of efficiency.

As seen in the literature surveyed above, the value of flexibility is evident from an operational point of view (e.g. matching supply and demand). On the other hand, commitment can be valuable when it encourages a certain behavior from another player. In our context, the efficiency gains of flexibility are typically dominated by the reduced incentives for early production. Under a flexible subsidy policy, the government can get closer to a desired target sales, but the supplier extracts more surplus from the system. Therefore, a committed subsidy policy typically has a lower cost for the government.

1.3. Structure

The remainder of the paper is oulined as follows. In Section 2, we present the models for the government and the supplier. In Section 3, we solve the optimization problems and analytically compare the outcomes under the flexible and committed settings. We test these results with several of computational experiments in Section 4 and provide some concluding remarks in Section 5. All proofs are relegated to the Appendix.

2. Model

As we previously mentioned, we consider a dynamic Stackelberg game between the government and the supplier. The government is choosing a subsidy level to offer consumers at each period, denoted by r_t, followed by the supplier, who decides upon production quantities u_t. At the end of each period, the uncertain demand is realized and the remaining inventory (if any) is carried over to the next period. The two settings mentioned before, committed and flexible, differ only on the timing of the government's decision. Under a committed setting, the government sets subsidy levels for all periods before the horizon begins and commits to these subsidies. In the flexible setting, the subsidy levels are decided at the beginning of each period, possibly varying as a function of previous production quantities and realized demand levels.

In order to keep the analysis tractable and draw insights, we consider a two period horizon, $t \in \{1, 2\}$. The advantages of policy commitment versus flexibility should be evident even within this two period model. The intuition built for two periods can be expanded for longer horizons, as the different periods decouple given the state of the system, namely the left-over inventory and the realized sales level. For conciseness, we focus only on the two period setting.

Within these two time periods, the government aims to achieve an adoption target level Γ, in expectation. More precisely, the government's goal is to incentivize at least Γ consumers to adopt the technology by the end of the time horizon. This policy target is public information, known to consumers and the industry. For example, in the 2011 State of the Union address, US President Barack Obama mentioned the following goal: "With more research and incentives, we can break our dependence on oil with bio-fuels and become the first country to have a million electric vehicles on the road by 2015"[3]. Another example of such adoption target has been set for solar panels in the California Solar Incentive (CSI) program, which states by 2016 "a goal to install approximately 1,940 MW of new solar generation capacity"[4]. Hence, in our model, we optimize the subsidy level to achieve a given adoption target level while minimizing government expenditure.

In order to achieve this target adoption, the government sets consumer subsidy r_t, for each time period t. Any consumer who purchases the product at that time period will be awarded that subsidy. At each period $t \in \{1, 2\}$, the supplier chooses production quantities u_t as a function of the current level of inventory x_t and the subsidy levels r_t announced by the government. The number of available units to be sold at each period is given by: $Supply_t = x_t + u_t$.

Demand for the product at time t is realized as a function of the subsidy levels r_t and the nominal uncertain demand ϵ_t. The random variable ϵ_t represents the intrinsic demand for the product if no subsidy was offered (i.e., $r_t = 0$). This intrinsic demand ϵ_t is assumed to have a probability distribution that is known by both the government and the supplier. Assume that for each additional subsidy dollar in r_t, we obtain an additional b_t units of demand. This value b_t is the demand sensitivity at time t with respect to the subsidy. Demand can be formally defined as: $Demand_t = b_t r_t + \epsilon_t$.

The sales level s_t will be determined by the subsidy level, the production decisions of the supplier and the uncertainty realization. Given a supply level and a demand realization at time t, the number

[3] Department of Energy - "One Million Electric Vehicles By 2015" - Status Report - February 2011

[4] California Solar Incentive Program - http://www.gosolarcalifornia.ca.gov/about/csi.php

of units sold s_t is the minimum of supply and demand, that is: $s_t = \min(Supply_t, Demand_t) = \min(x_t + u_t, b_t r_t + \epsilon_t)$. The inventory left for the next period can be expressed as $x_{t+1} = x_t + u_t - s_t$.

The objective of the government is to minimize total expected spending while still satisfying the adoption sales target, Γ, in expectation. More precisely, in our two period model, the government's objective is to minimize $E[Spending] = E[r_1 s_1 + r_2 s_2]$ subject to an average sales target constraint: $E[Sales] = E[s_1 + s_2] \geq \Gamma$.

The subsidy optimization model with an adoption target described above is not the only possible model for the government. For example, one may consider other target constraints on the distribution of sales. Alternatively, the government could maximize sales or social welfare with a budget constraint (see for example Taylor and Xiao (2014b) and Alizamir et al. (2012)). As we show later in our model, the government reacts to early low sales by increasing the subsidy level in later periods. This creates what we call the *undersupply incentive*. Among our main results in this paper, we show that flexibility is typically costlier for the government because of this incentive. Any alternative model for the government problem where the subsidy increases when early sales are low should still create this undersupply incentive for the firm. For simplicity, we focus this paper on the expected sales level constraint model, but note that alternative government constraints should yield qualitatively similar results.

The supplier seeks to maximize the total expected profits by choosing production levels u_t. There is a fixed linear production cost c_t for each unit produced, u_t. The unit selling price p_t is assumed to be exogenous and fixed before the beginning of the time horizon. Units not sold by the end of the horizon ($t = 2$) get sold for a salvage value denoted by p_3. More formally, the supplier's objective can be written as: $E[Profit] = E[p_1 s_1 - c_1 u_1 + p_2 s_2 - c_2 u_2 + p_3 x_3]$. In summary, the two players are solving the following optimization problems.

Government	Supplier
$\min_{r_1, r_2 \geq 0} E[Spending]$ $E[Sales] \geq \Gamma$	$\max_{u_1, u_2 \geq 0} E[Profit]$

We focus this paper on a single supplier, which can be seen as an aggregate industry player. If we assume there are multiple symmetric suppliers and the aggregate demand is split deterministically across all firms, Lippman and McCardle (1997) show that there is a unique equilibrium to the competitive single-period newsvendor game. Furthermore, this equilibrium is symmetric and the aggregate order level is the same as the monopolistic setting. Using that same logic in our dynamic model, all the results of this paper can be derived for the symmetric competitive setting. Looking at the single supplier as an aggregate industry player further motivates the exogenous price that is not controlled by a given firm. We present the model in this paper using a single supplier to simplify the exposition.

As mentioned before, the order of decisions is the key difference between the two settings we want to study: committed and flexible. In the committed setting, the government commits to subsidy levels r_1 and r_2 for both consecutive periods. The supplier then decides the first production quantity u_1 and the first period nominal demand ϵ_1 is realized. Observing the amount of inventory x_2 left after the first period, the supplier decides the second production quantity u_2. The second demand ϵ_2 is then realized. In the flexible setting, the government chooses only the first subsidy level r_1. The supplier then follows by choosing a production quantity u_1 and the first period demand ϵ_1 is realized. At the end of the first period, the government sets the subsidy level for the second period, r_2, followed by the supplier's decision u_2 and the demand realization ϵ_2. The sequence of events describing these two settings is displayed in Figure 1. We use the superscripts c and f to represent the committed and flexible settings respectively. We next present in more details the dynamic programs for each setting.

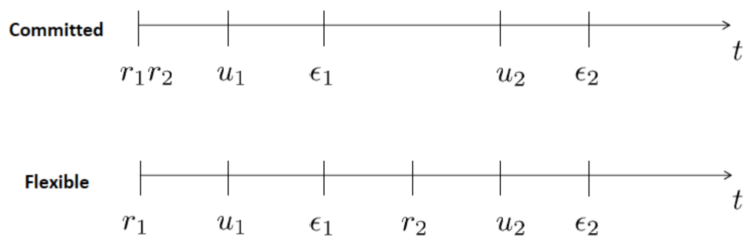


Figure 1 Sequence of events under committed and flexible settings.

2.1. Committed setting

In the committed setting, the government leads the game by choosing both subsidy levels and the supplier follows by deciding production quantities. The optimal decisions by each party can be viewed as a dynamic optimization problem. In the first stage, the government chooses a subsidy policy r_1 and r_2 subject to the optimal production policy set by the supplier. The optimal supplier policy can be expressed as the solution to a two-stage profit maximization problem, for given values of r_1 and r_2.

Let $h_2^c(x_2, r_2, \epsilon_1)$ denote the second period profit-to-go of the supplier under the committed setting, given the current inventory level x_2 and the demand realization ϵ_1. We do not assume ϵ_1 and ϵ_2 to be independent, therefore the first period demand realization is part of the state-space in the dynamic optimization.

In the first period, the manufacturer solves the following problem to maximize the expected first period profit plus the profit-to-go for the second period. Note that the effect of the first production decision u_1 on the profit-to-go is captured by the inventory x_2. This quantity is given by the supply level in the first period minus the sales: $x_2 = x_1 + u_1 - \min(x_1 + u_1, b_1 r_1 + \epsilon_1)$. The optimal objective value of this optimization problem is defined as the optimal expected profit of the supplier and given by:

$$h_1^c(r_1, r_2) = \max_{u_1 \geq 0} E_{\epsilon_1}[p_1 \min(x_1 + u_1, b_1 r_1 + \epsilon_1) - c_1 u_1 + h_2^c(x_2, r_2, \epsilon_1)] \tag{1}$$

At the beginning of the second period, the manufacturer solves problem (2) to maximize the second period expected profit that includes the remaining salvage value. This problem also defines the profit-to-go function used in the first period optimization in (1):

$$h_2^c(x_2, r_2, \epsilon_1) = \max_{u_2 \geq 0} E_{\epsilon_2|\epsilon_1}[p_2 \min(x_2 + u_2, b_2 r_2 + \epsilon_2) - c_2 u_2 + p_3 \max(x_2 + u_2 - b_2 r_2 - \epsilon_2, 0)] \tag{2}$$

The objective function above is composed of the second period expected revenue, minus production cost, plus the expected salvage value for left-over inventory at the end of the horizon.

We define $u_1^c(r_1, r_2)$ and $u_2^c(x_2, r_2, \epsilon_1)$ to be the optimal production quantities under the committed setting, which are the optimal solutions of problems (1) and (2) respectively, as a function of the subsidy levels r_1 and r_2. Given the supplier's best-response policy, the government's objective is to minimize the expected spending, subject to a target adoption constraint. The government problem under the committed setting is given by:

$$\begin{aligned} E[\mathit{Spending}^c] = \min_{r_1, r_2 \geq 0} \; & E[r_1 \min(x_1 + u_1^c(r_1, r_2), b_1 r_1 + \epsilon_1) + r_2 \min(x_2 + u_2^c(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \\ \text{s.t.} \; & E[\min(x_1 + u_1^c(r_1, r_2), b_1 r_1 + \epsilon_1) + \min(x_2 + u_2^c(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \geq \Gamma \end{aligned} \tag{3}$$

The optimal solution to problem (3) defines the optimal subsidy levels r_1^c and r_2^c and the optimal expected spending level $E[\mathit{Spending}^c]$ under the committed setting. The expected profit of the supplier is defined as $E[\mathit{Profit}^c] = h_1^c(r_1^c, r_2^c)$. The total sales under the optimal subsidy levels is defined as $\mathit{Sales}^c = \min(x_1 + u_1^c(r_1^c, r_2^c), b_1 r_1^c + \epsilon_1) + \min(x_2 + u_2^c(x_2, r_2^c, \epsilon_1), b_2 r_2^c + \epsilon_2)$.

2.2. Flexible setting

In the flexible setting, the government leads the game by choosing only the first period subsidy level. The supplier follows by choosing the production quantity for the first period and then the game is repeated for the second period. The optimal decisions by each party can be viewed as a multi-tiered optimization problem. In the first stage, the government chooses a subsidy policy r_1 anticipating the optimal response of the supplier, u_1. That production quantity, u_1, is decided by the supplier while considering the government's policy for the second period subsidy r_2, which is itself a function of the sales in the first period.

From the supplier's perspective, the state of the system at the second period is composed of the leftover inventory, x_2, the subsidy level, r_2 and the demand realization, ϵ_1. Note that ϵ_1 can have some information about the next demand realization ϵ_2, as we consider a correlated demand model. For any given state, define $h_2^f(x_2, r_2, \epsilon_1)$ as the profit-to-go function of the supplier at period $t = 2$ under the flexible setting.

From the government's perspective, the state of the system at the second period is composed of the sales from the first period, s_1, the leftover inventory of the supplier, x_2, and the demand realization, ϵ_1. The first period sales captures information about how far is the government from its target level Γ. The inventory level affects the possibility of a stock-out and the previous demand realization may influence future demand. Knowing the strategy of the supplier, the government can set the second subsidy level r_2 that minimizes the cost of achieving the remaining target. We denote by $g^f(x_2, s_1, \epsilon_1)$ the second period cost-to-go of the government.

Because of the sequential nature of the dynamic problem for the flexible setting, we first formulate the optimization problems for the second period:

$$h_2^f(x_2, r_2, \epsilon_1) = \max_{u_2 \geq 0} E_{\epsilon_2|\epsilon_1}[p_2 \min(x_2 + u_2, b_2 r_2 + \epsilon_2) - c_2 u_2 + p_3 \max(x_2 + u_2 - b_2 r_2 - \epsilon_2, 0)] \tag{4}$$

Define $u_2^f(x_2, r_2, \epsilon_1)$ as the optimal second period production policy under the flexible setting, which is the optimal solution to problem (4). The government problem in the second period can be written as follows:

$$\begin{aligned} g^f(x_2, s_1, \epsilon_1) = \min_{r_2 \geq 0} \; & E_{\epsilon_2|\epsilon_1}[r_2 \min(x_2 + u_2^f(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \\ \text{s.t.} \quad & s_1 + E_{\epsilon_2|\epsilon_1}[\min(x_2 + u_2^f(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \geq \Gamma \end{aligned} \tag{5}$$

Define $r_2^f(x_2, s_1, \epsilon_1)$ as the optimal second period subsidy under the flexible setting, which is the optimal solution to problem (5). Knowing the government's future response in subsidy policy, the supplier can decide its first period production level by solving the following optimization problem:

$$\begin{aligned} h_1^f(r_1) = \max_{u_1 \geq 0} \; & E_{\epsilon_1}\left[p_1 s_1 - c_1 u_1 + h_2^f\left(x_1 + u_1 - s_1, r_2^f(x_1 + u_1 - s_1, s_1, \epsilon_1), \epsilon_1\right)\right] \\ & \text{where } s_1 = \min(x_1 + u_1, b_1 r_1 + \epsilon_1) \end{aligned} \tag{6}$$

Note that we use s_1 as a shorthand notation for first period sales, which should not be confused as an optimization constraint. The optimal first period order quantity $u_1^f(r_1)$, should maximize both the immediate expected profit plus the expected second period profit-to-go. Knowing the contingent production strategy of the supplier, $u_1^f(r_1)$, the government must then find the optimal first period subsidy r_1^f that minimizes both the immediate cost and the second period cost-to-go.

$$\begin{aligned} E[Spending^f] = \min_{r_1 \geq 0} \; & E[r_1 s_1 + g^f(x_1 + u_1^f(r_1) - s_1, s_1, \epsilon_1)] \\ & \text{where } s_1 = \min(x_1 + u_1^f(r_1), b_1 r_1 + \epsilon_1) \end{aligned} \tag{7}$$

By sequentially solving problems (4) to (7), one can obtain the optimal decision variables for both the supplier and the government under the flexible setting. The expected government spending $E[Spending^f]$ is defined in (7). From (6), we define the supplier's expected profit under the optimal subsidy: $E[Profit^f] = h_1^f(r_1^f)$.

3. Impact on Government and Supplier

In this section, we solve the dynamic programming formulations for both the committed and flexible settings and characterize the optimal decision variables. Then, we compare the outcomes in both settings for the government, the supplier and the consumers.

3.1. Optimal subsidy and production levels

In order to maintain the analysis tractable when solving problems (1) through (7), we impose a few assumptions on the model parameters, which we argue are reasonable for markets with developing technologies. The first assumption relates to demand correlation across time periods. Dynamic games are often studied with independent shocks, but this would remove one of the key benefits of flexibility, which is adapting to new demand information. In this paper, we consider a more general model that allows positive or negative correlation across time periods. In particular, we assume that a random shock from the first period demand can linearly affect the second period demand. This model is used in the literature for various applications (see e.g., See and Sim (2010)). The nominal demand model we consider is summarized in the following assumption.

Assumption 1. *Define the nominal demand ϵ_t at time $t \in \{1,2\}$ by:*

$$\epsilon_1 = \mu_1 + w_1$$
$$\epsilon_2 = \mu_2 + \alpha w_1 + w_2$$

$\mu_t > 0$ is the average demand at time t. The random shocks w_1 and w_2 are independent random variables with zero mean: $E[w_1] = E[w_2] = 0$. We denote the cumulative distribution function (CDF) of w_t by the continuous function $F_t(\cdot)$, which is assumed to be common knowledge for both the government and the supplier. In addition, the random variables w_t are assumed to have bounded supports, $w_t \in [A_t, B_t]$ such that the nominal demands are non-negative, i.e., $\mu_1 + A_1 \geq 0$ and $\mu_2 + \min(\alpha A_1, \alpha B_1) + A_2 \geq 0$.

Note that the CDFs $F_t(\cdot)$ do not need to be identical across time periods. The parameter α represents the level of correlation between time periods (α can be either positive or negative). More precisely, the correlation coefficient between ϵ_1 and ϵ_2 is given by: $Corr(\epsilon_1, \epsilon_2) = \alpha\sqrt{\frac{Var(w_1)}{Var(w_2)}}$.

In early stages of the introduction of new technologies, it is often common to observe decreasing prices and costs over time. In addition, profit margins are often decreasing over time, as additional players are entering the market. With this in mind, we restrict our analysis with the following set of inequalities summarized in Assumption 2. Note that in our model, the supplier is a price-taker, so that p_1, p_2 and p_3 are exogenous market prices (p_3 being the salvage value at the end of the horizon). The marginal costs of production are denoted by c_1 and c_2.

Assumption 2. *We make the following assumptions on prices, costs and profit margins:*

1. *Prices and costs are decreasing over time, i.e.,* $p_1 > p_2$ *and* $c_1 > c_2$.
2. *Profit margins are positive and decreasing, i.e.,* $p_1 - c_1 > p_2 - c_2 > 0$.
3. *Salvage value is smaller than production cost:* $c_2 > p_3$.

Decreasing prices and costs are commonly observed in the literature for new product introduction. Lobel and Perakis (2011), for instance, surveys the literature on the declining costs of solar photovoltaic technology, mostly attributed to learning effects. Lee et al. (2000) show additional evidence of declining prices in the PC industry within the product life-cycle. Note that cost decreases are often attributed to learning-by-doing, which could be modeled endogenously as a function of units sold or produced. As we will see later, the committed setting already has an advantage to encourage higher supply levels. In this case, endogenous learning might give further advantage to the committed setting. In order to simplify the problem and to focus solely on the impact of the game dynamics, we assume the production cost reduces exogenously.

For the same reason, we restrict our model to the case with decreasing profit margins. If profit margins were to increase, we would provide further incentives for the supplier to delay production. The production delay would be more accentuated in the flexible setting, making a stronger case for policy commitment.

We next define in Table 1 a set of quantiles of the cumulative distribution of demand uncertainty, $F_t(\cdot)$. We later show in Lemma 1 that these quantities represent the optimal production quantiles of the supplier in the different periods and settings.

Table 1 **Production quantiles**

Committed	Flexible
$k_1^c = F_1^{-1}\left(\frac{p_1-c_1}{p_1-c_2}\right)$	$k_1^f = F_1^{-1}\left(\frac{(p_1-c_1)-(p_2-c_2)}{p_1-p_2}\right)$
$k_2^c = F_2^{-1}\left(\frac{p_2-c_2}{p_2-p_3}\right)$	$k_2^f = F_2^{-1}\left(\frac{p_2-c_2}{p_2-p_3}\right)$

Note that the production quantiles for the second period are the same in both setting $k_2^c = k_2^f = k_2$. In addition, observe that $k_1^f \le k_1^c$. Before showing the optimality of the production quantiles from Table 1, we impose an additional assumption. More precisely, we restrict our attention to the case where the supplier does not stay idle at any of the time periods. This happens when the left-over inventory is smaller than the desired supply level for the next period. Otherwise, the optimal ordering policy would have a discontinuity that makes the problem analytically intractable in the first period. Realistically, green technology products are expensive to manufacture and typically don't face a critical oversupply where the left-over inventory from one year covers all demand for the next year. For this reason, we restrict the magnitude of the demand noise so that the inventory x_2 should be no larger than the desired supply level at period 2 for any realization of w_1. We also restrict our attention to the case where the adoption target cannot be reached without the presence of government subsidies. We summarize this discussion in the following assumption.

ASSUMPTION 3. *On the magnitude of demand uncertainty and adoption target:*

1. *Desired supply at* $t=1$ *is always larger than initial inventory, i.e,.* $k_1^f + \mu_1 \geq x_1$.

2. *Desired supply at* $t=2$ *is always larger than leftover inventory, i.e.,* $k_2 + \mu_2 \geq k_1^c - A_1 - \min(\alpha A_1, \alpha B_1)$.

3. *The adoption target is large enough, i.e.,* $\Gamma \geq 2(E[\min(k_1^c, w_1)] + \mu_1)$ *and* $\Gamma \geq 2(E[\min(k_2^c, w_2)] + \mu_2)$.

Assumption 3 is not necessary, but sufficient to guarantee that the supplier will not idle. Note that in the first part, we use k_1^f, whereas in the second part, we use k_1^c. Since $k_1^f \leq k_1^c$, this ensures that both conditions are satisfied under both settings. The first part, $k_1^f + \mu_1 \geq x_1$, guarantees the first period production level is non-negative. Indeed, if the initial inventory is too large, the problem becomes uninteresting. The second part means the target "newsvendor" service level of the second period is larger than in the first period. In other words, in the absence of a subsidy policy, the manufacturer would try to serve a larger number of customers in the second period simply from demand, cost and price conditions. The last part of the assumption ensures that the government subsidy policy is actually needed to meet the target adoption. In other words, we want to restrict our model with Assumption 3 to ensure that $r_t^j > 0$ and $u_t^j > 0$ for any period t and setting j.

Under Assumptions 1, 2 and 3, one can obtain the optimal production policies for the supplier in each setting. The results are derived in closed form and summarized in Lemma 1.

LEMMA 1. *Optimal ordering production and subsidy levels for both settings.*

- *The optimal ordering quantities for the supplier in both settings are given by:*

Committed	Flexible
$u_1^c(x_1, r_1) = b_1 r_1 + k_1^c + \mu_1 - x_1$	$u_1^f(x_1, r_1) = b_1 r_1 + k_1^f + \mu_1 - x_1$
$u_2^c(x_2, r_2, w_1) = b_2 r_2 + k_2 + \mu_2 + \alpha w_1 - x_2$	$u_2^f(x_2, r_2, w_1) = b_2 r_2 + k_2 + \mu_2 + \alpha w_1 - x_2$

- *The optimal subsidy levels are given by:*

Committed	Flexible
$r_1^c = \dfrac{\Gamma}{b_1+b_2} - \dfrac{(v_1^c+\mu_1)(2b_1+b_2)}{2b_1(b_1+b_2)} - \dfrac{v_2+\mu_2}{2(b_1+b_2)}$	$r_1^f = \dfrac{\Gamma}{b_1+b_2} - \dfrac{(v_1^f+\mu_1)(2b_1+b_2)}{2b_1(b_1+b_2)} - \dfrac{v_2+\mu_2}{2(b_1+b_2)}$
$r_2^c = \dfrac{\Gamma}{b_1+b_2} - \dfrac{(v_2+\mu_2)(b_1+2b_2)}{2b_2(b_1+b_2)} - \dfrac{v_1^c+\mu_1}{2(b_1+b_2)}$	$r_2^f(s_1, x_2, w_1) = \dfrac{\Gamma - s_1 - \mu_2 - \alpha w_1 - v_2}{b_2}$

Note that the above ordering quantities are functions of the subsidy levels as they are computed as best responses. Note also that the optimal supply level at time t, $u_t + x_t$, is expressed as the nominal demand level, plus the demand boost from the subsidy $b_t r_t$, adjusted by the newsvendor quantile k_t. With this optimal production policy, one can solve the government optimization problem and obtain the optimal subsidy policy. To simplify the notation, we denote by v_t^j the expected demand uncertainty truncated by the optimal quantile. That is, $v_t^j = E[\min(k_t^j, w_t)]$, for setting $j \in \{c, f\}$ and time period $t \in \{1, 2\}$. Note that $v_2^f = v_2^c = v_2$.

One can see that the optimal subsidy levels for the second period under the committed setting $r_2^f(s_1, x_2, w_1)$ is a random variable that depends on the realization of w_1. For comparison purposes, it can be useful to compute the corresponding expected value. We first compute the expected sales at the first period: $E[s_1] = b_1 r_1^f + \mu_1 + v_1^f$. Consequently, the expected subsidy level for the second period in the flexible setting is given by:

$$E[r_2^f(s_1, x_2, w_1)] = \frac{\Gamma}{b_1 + b_2} - \frac{(v_2 + \mu_2)(b_1 + 2b_2)}{2b_2(b_1 + b_2)} - \frac{v_1^f + \mu_1}{2(b_1 + b_2)}$$

3.2. Comparisons

When comparing the flexible and committed settings, the first thing to notice is the difference in the optimal quantiles for any given subsidy levels. To provide further intuition for the optimality of the ordering quantiles described in Table 1, one can look at the cost of underage and overage in traditional newsvendor models. Note that the key difference between k_1^c and k_1^f is the cost of undersupplying the market demand. In the single-period newsvendor model, the cost of underage (C_u) and overage (C_o) define the optimal ordering quantile as $F^{-1}(\frac{C_u}{C_u + C_o})$. Since there is no idling in the second period, an additional unsold unit (overage) will incur a cost that is simply the difference in production cost over time. For both settings the cost of overage is defined as $C_o = c_1 - c_2$. For the committed setting, the underage cost is defined by the opportunity cost, or profit margin forgone, $C_u^c = p_1 - c_1$. In the flexible case, an unmet unit of demand will be compensated by an equivalent unit of demand from increased rebates in the second period. Therefore the underage costs is the difference in profit margins, $C_u^f = (p_1 - c_1) - (p_2 - c_2)$. Note that the quantiles of Table 1 are also defined by the rule $\frac{C_u}{C_u + C_o}$. The proof of Lemma 1 contains a formal proof for this optimality result. Nevertheless, this explanation brings a very interesting intuition: government flexibility reduces the underage risk for the supplier.

This key difference in the ordering levels is further described in Proposition 1. It drives disparities in production, subsidies and sales in the two settings. Note that sales at a given period t are defined as $s_t = \min(x_t + u_t, b_t r_t + \epsilon_t)$ and is a random variable. With the structure of the optimal production policy defined in Lemma 1, note that sales for each setting $j \in \{c, f\}$ can be simplified to $s_1^j = b_1 r_1^j + \mu_1 + \min(k_1^j, w_1)$ and $s_2^j = b_2 r_2^j + \mu_2 + \alpha w_1 + \min(k_2^j, w_2)$. The following proposition summarizes these comparisons.

Proposition 1. *Comparing production quantiles, expected productions, subsidy levels and sales between committed and flexible settings:*

- *In the first period, the supplier's optimal production quantile is larger in the committed setting than in the flexible setting, i.e.,* $k_1^c \geq k_1^f$*. In the second period, the quantiles are equal:* $k_2^c = k_2^f = k_2$*.*
- *The expected production is larger at the first period and lower at the second period in the committed setting. The total expected production is the same in both settings, i.e.,* $u_1^c + E[u_2^c] = u_1^f + E[u_2^f]$*.*
- *Expected subsidy levels at each period are lower in the committed setting.*
- *The expected sales are higher in the first period with commitment, but lower in the second period. Also, the total expected sales meets the government target in both settings, i.e.,* $E[s_1^c + s_2^c] = E[s_1^f + s_2^f] = \Gamma$*.*

Production	subsidy	Sales
$u_1^c \geq u_1^f$	$r_1^c \leq r_1^f$	$E[s_1^c] \geq E[s_1^f]$
$E[u_2^c] \leq E[u_2^f]$	$r_2^c \leq E[r_2^f]$	$E[s_2^c] \leq E[s_2^f]$

Note that the subsidy levels and the production quantities at the first period are not random variables. The fact that the total expected sales are equal to the target adoption level is not surprising, as the government uses this condition to derive the optimal solution. Proposition 1 shows that a larger proportion of the target is satisfied in the first period under commitment. To show this, one can calculate the difference in sales quantity: $E[s_1^c - s_1^f] = \frac{b_2}{2(b_1+b_2)}(v_1^c - v_1^f)$. This measure quantifies the average amount of sales that is postponed to the second period when the game dynamics is changed from a committed to a flexible setting.

In order to understand the effect of this postponement on the total government spending, we need to further analyze the optimal subsidy levels. Using the results from Proposition 1, one can compare the expected level of spending from the government. Under a committed setting, the spending will be given by $E[Spending^c] = E[s_1^c]r_1^c + E[s_2^c]r_2^c$, as subsidy levels are set in a deterministic way. Under a flexible setting, the spending is defined as $E[Spending^f] = E[s_1^f]r_1^f + E[s_2^f r_2^f]$. Note that the subsidy for the second period under the flexible setting is now a random variable and therefore cannot be taken outside the expectation. We next derive the expected total spending levels for the government under the two settings.

Theorem 1. *The expected government spending is given by:*

$$E[Spending^c] = (b_1 r_1^c + v_1^c + \mu_1)r_1^c + (b_2 r_2^c + v_2^c + \mu_2)r_2^c$$
$$E[Spending^f] = (b_1 r_1^f + v_1^f + \mu_1)r_1^f + (b_2 E[r_2^f] + v_2^f + \mu_2)E[r_2^f] + \frac{Var(\min\{k_1^f, w_1\})}{b_2} + \frac{\alpha E[w_1 \min\{k_1^f, w_1\}]}{b_2}$$

The difference in expected spending between the two settings can be written as:

$$E[Spending^f - Spending^c] = \overbrace{\frac{1}{4b_1(b_1+b_2)}\left[2b_1(v_1^c - v_1^f)(2\Gamma - v_2^c - \mu_2) + b_2(v_1^c+\mu_1)^2 - b_2(v_1^f+\mu_1)^2\right]}^{\text{supplier incentive effect}} + \underbrace{\frac{Var(\min\{k_1^f, w_1\})}{b_2}}_{\text{adaptability effect}} + \underbrace{\frac{\alpha E[w_1 \min\{k_1^f, w_1\}]}{b_2}}_{\text{correlation effect}}$$

Corollary 1. *If $\alpha \geq 0$, the expected spending is smaller in the committed setting relative to the flexible setting, i.e., $E[Spending^c] \leq E[Spending^f]$.*

Note that the difference in spending between committed and flexible is derived by different effects, which we label: *supply incentive; adaptability; correlation.* The first term is caused by the reduced production quantile, $k_1^f < k_1^c$. Since government flexibility reduces the firm's potential loss from undersupply, we label this the *supply incentive effect.* This effect captures the cost for the flexible government to compensate for the reduced supply.

The second term, *adaptability effect*, captures the average premium paid by the government for the benefit of adjusting the rebates in the second period. Even in the absence of the supply incentive effect, $k_1^f \to k_1^c$, and the correlation effect, $\alpha = 0$, the adaptability effect on the flexible spending will remain solely due to volatility in first period demand. This effect occurs because when the first demand is low, the second period subsidy increases together with sales. When first period demand is high, the positive upside is curbed by the limited supply level k_1^f. This adaptability will effectively "buy" the government a lower variance in sales, as shown in Theorem 2 below.

The third term, *correlation effect*, appears when there is inter-temporal correlation in demand, $\alpha \neq 0$. Note that the two first effects are always positive. When correlation is non-negative, Corollary 1 shows that since the third effect is also positive, the committed spending is on average smaller. When α is sufficiently negative, the correlation effect can become the dominant effect and make the expected flexible spending lower than the committed. This instance is demonstrated in the computational experiments of Section 4.

The following result compares the variance of total sales realized under the flexible and the committed setting. Note that the expected sales in both cases equal the adoption target. We show in Theorem 2 below that the total output of sales is more variable under the committed setting.

Theorem 2. *The variance of the sales is larger in the committed setting relative to the flexible setting, i.e., $Var(Sales^c) \geq Var(Sales^f)$.*

In other words, the premium paid for adaptability in expected spending provides a lower variance in sales. The flexible government will typically reach a final adoption level closer to the desired target. This result holds any level of demand correlation. Note that this is not variance in spending, which can be significantly more complicated to compare analytically. We do compare the variance of spending computationally in Section 4, where we show that there is no clear dominance between the two settings. In fact, we show how it depends on the market conditions.

In the absence of correlation, the variance in sales and expected spending characterize a risk-reward tradeoff for the government. Depending on how close they want to be to the adoption target, the government might consider paying the premium for a flexible policy.

Next, we compare the expected supplier's profit in both settings.

THEOREM 3. *The expected profit of the supplier is smaller in the committed setting relative to the flexible setting, i.e.,* $E[Profit^c] \leq E[Profit^f]$.

Theorem 3 states that the supplier will always profit more under a flexible government. This result is caused by lower opportunity cost of undersupply provided by the government in the flexible case. This lower cost of undersupply will lead to the lower production quantile, k_1^f, which leads to higher average subsidy levels.

It should be noted that this is not a direct manipulation of the government policy by the supplier. In the absence of demand uncertainty or when $p_2 - c_2 << p_1 - c_1$, the flexible subsidy level converges to the committed level and the profit difference goes to zero. The game dynamics of the flexible setting does not provide an additional profit for the firm by itself. The firm also does not hold any informational advantage over the government. The additional profit in the flexible setting comes from the undersupply incentive created by the government policy that boosts demand in the second period if initial sales are low.

3.3. Consumer Surplus

In a deterministic demand model, consumer surplus is typically defined as $D^2/2b$. The definition in equation (8) below adjusts for the fact that there is a stock-out probability that affects the utility of consumers. The underlying assumption is that every consumer has the same probability of being not served, independent of their individual valuation for the product. For more details, see for example Cohen et al. (2013) for a definition of consumer surplus under stochastic demand. We define the consumer surplus for one time-period as:

$$CS(\epsilon) = \frac{D(\epsilon)\min(u + x, D(\epsilon))}{2b} \tag{8}$$

Here, u represents the production quantities and x the left over inventory from the previous period. The total consumer surplus is obtained as the sum of each period's consumer surplus:

$$\begin{aligned} CS &= CS_1(\epsilon_1) + CS_2(\epsilon_2) \\ CS &= \frac{(b_1 r_1 + \epsilon_1)\min(u_1 + x_1, b_1 r_1 + \epsilon_1)}{2b_1} + \frac{(b_2 r_2 + \epsilon_2)\min(u_2 + x_2, b_2 r_2 + \epsilon_2)}{2b_2} \end{aligned}$$

Note that the consumer surplus is a random variable that depends on both noises. In order to compare both settings, we look at the expected value of the consumer surplus. In particular, we want to know when consumers are better off in the flexible setting and what are the key factors. We focus on the uncorrelated case for simplicity, i.e., $\alpha = 0$.

We have shown that in the flexible setting, the subsidies are higher in both periods in expectation. Therefore, the consumers are receiving more money on average per unit sold. In addition, the expected total productions are the same. Therefore, one might naively think that consumers are always better off in the flexible policy. However, this is not always the case and the expected total consumer surplus can be larger in the committed setting under some conditions. The results are summarized in the following Theorem.

THEOREM 4. *The expected consumer surplus satisfies the following.*

1. *In the second period, the consumers are always better off in the flexible setting, i.e.,*

$$E[CS_2^f] \geq E[CS_2^c]. \tag{9}$$

2. *In the first period, consumers can be better off or worse off in the flexible setting, depending on the ratio of price sensitivities. In particular, if $\Gamma \geq 2w_1$, there exists a threshold value of b_1/b_2 above (below) which, the consumers are better off in the flexible (committed) setting.*

The consumers are mainly affected by the amount of subsidies offered by the government and by the total sales (availability of product). We note that in the second time period, both the expected subsidies and sales are larger in the flexible setting. Consequently, it overall benefits consumers and the result in (9) is intuitive. On the other hand, the impact on consumers in the first period is more complicated. Indeed, the subsidies in the flexible setting are higher but the expected sales are lower. As a result, the effect on consumers depends on the tradeoff between these two factors. In particular, we show that it depends on the price sensitivity parameters ratio b_1/b_2. If this ratio is large enough, consumers are better off in the flexible setting and if this ratio is small enough, consumers are worse off. The assumption $\Gamma \geq 2w_1$ is a sufficient condition for the existence of the b_1/b_2 threshold. It is a technical conditional that is not very restrictive, as it only ensures the target adoption level set by the government cannot be attained simply by a large noise realization.

Finally, one can expect the total expected consumer surplus, $E[CS_1] + E[CS_2]$, to behave in a similar way as the the expected consumer surplus in the first period. In particular, there exists a threshold value of b_1/b_2 above which, the consumers are better off in the flexible setting (or worse below the threshold).

4. Computational Experiments

In this section we develop a numerical experiment to illustrate the impact of varying profit margins and demand uncertainty on the government spending and supplier's profit level. The numbers used for these simulations are based on the German solar photovoltaic market. Further details on the calculations used to develop the computational experiments can be found in Appendix 6.7. In summary, the data input for this simulation consists of: the government adoption target and the basic market parameters such as price, cost, average nominal demand (in the absence of rebate), demand sensitivity to rebates, and salvage value. Some of these parameters used are based on historical figures, while others are roughly estimated (such as the demand sensitivity). To demonstrate the effects of market conditions on committed/flexible policies, the second period costs and variance of the demand uncertainty are chosen at various levels. The demand uncertainty is drawn from a uniform distribution. Experiments with other distributions have yielded the same qualitative results, therefore they will not be displayed. Finally, we vary the degree of demand correlation across time periods to illustrate its impact on expected spending. It should be made clear that the data used in this section is used only as a basis for the simulation, which is meant to develop intuition about our model and is not an empirical investigation.

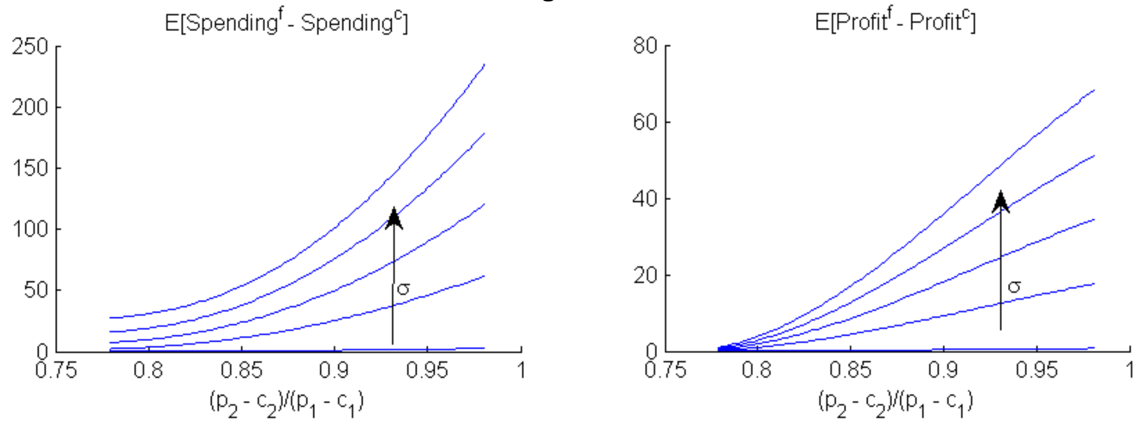
In the simulations presented in Figures 2 and 3, spending and profits are displayed in these figures in millions of €. Sales are measured in MW's of installed solar panels. The adoption target used to base this simulation was the $7500MW$ sold in Germany during the year of 2011. For Figures 2 and 3 we assume there is no inter-temporal correlation in demand, $\alpha = 0$. Correlation is introduced later in Figure 4.

In Figure 2, we observe the difference between the flexible and the committed settings in expected government spending and supplier's profit. The horizontal axis displays the level of the firm's second period profit margin, relative to the first. The vertical axis is the difference expected spending (left graph) and expected profit (right graph). A few observations are in order.

Observation 1: Between the two settings, the difference in expected spending, as well as supplier profit, converge to zero when demand uncertainty decreases, $\sigma \to 0$. This is to be expected, since the three effects displayed in Theorem 1 disappear without demand uncertainty.

Observation 2: When the profit margin of the second period is much smaller than the first $(p_2 - c_2)/(p_1 - c_1) \to 0$, the difference in profit for the supplier between the two settings also goes to zero. This occurs because the *supply incentive effect* disappears This can be largely explained by the convergence of the ordering quantiles k_1^c and k_1^f. When the second period sales are not very profitable, the underage cost of the supplier in the flexible case is not effectively mitigated by increased demand in the second period. With less incentive to undersupply from the industry side, the amount of subsidies needed from the government get closer to the committed case. Note

Figure 2



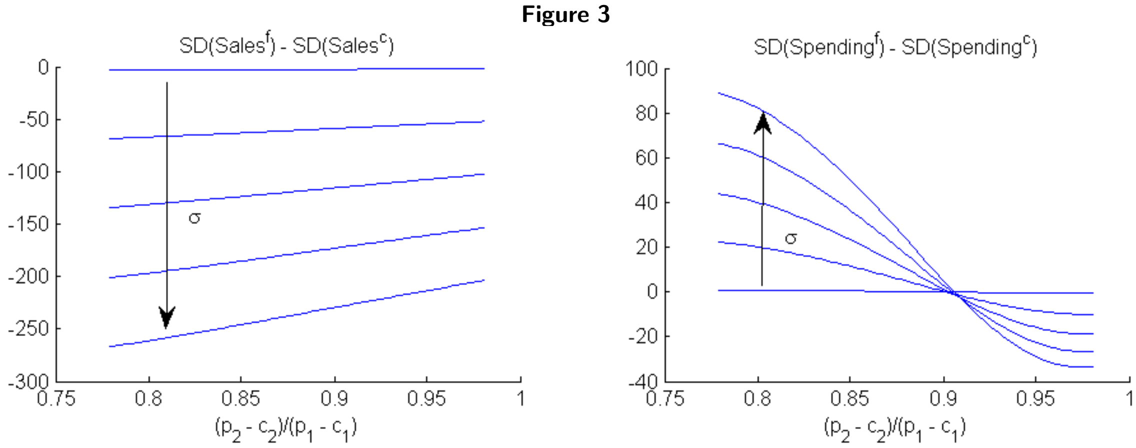
that the *correlation effect* is also absent in this example, since $\alpha = 0$. The remaining difference in spending converges to the remaining *adaptability effect*: $\frac{VAR(\min\{k_1^f, w_1\})}{b_2}$. This term decreases with the magnitude of the demand uncertainty, or equivalently the standard deviation σ.

Observation 3: For the simulation, we assume there is a baseline feed-in-tariff of 0.25€$/kWh$ (based on residential electricity prices) that would lead to the nominal demand levels. In order to reach the desired 7500MW of installations, we introduce additional subsidies that cost the government somewhere between 870 and 915 million € in a committed setting (depending on profit margins and demand uncertainty). As shown in Figure 2, the expected spending in a flexible setting can be as high as 235 million € more than the committed spending, when demand uncertainty is large and the profit margin of the second period is close to the profit of the first period. In other words, the additional flexbility premium is close to 25% of the cost of the subsidy program under policy committment. This indicates that committment vs. flexibility should be a significant concern for policy-makers. It is important to reiterate that the numbers presented here are only used to show the potential impact of a flexible/committed policy and are not meant to be used to evaluate past policy decisions.

In Figure 3, we observe the difference in the standard deviation of sales and government spending between the flexible and the committed settings. As before, the horizontal axis displays the level of the firm's second period profit margin, relative to the first. The vertical axis is the difference in the standard deviation of sales, measured in MW of installed solar panels (left graph), and the standard deviation of spending measured in millions of € (right graph).

Observation 4: The variance in total sales is indeed smaller, as expected from Theorem 2. The higher expected spending is indeed lowering the variance in sales in the flexible setting, allowing the government to be closer to the adoption target. On the other hand, the variance of spending is not necessarily lower in the flexible setting. In fact, when the profit margin of the second period is

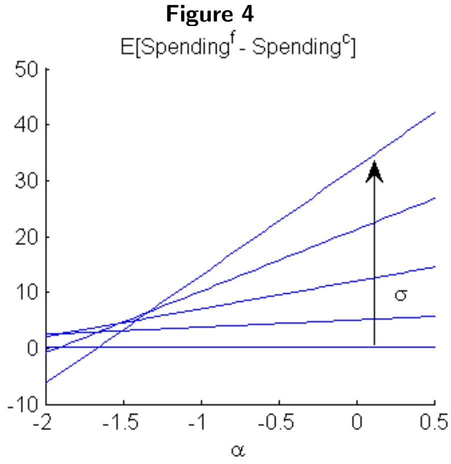
Figure 3



high enough, Figure 3 shows that the standard deviation in spending is lower in the flexible case. This is driven by the low variance in sales in the first period.

When the profit margin of the second period is too low, the standard deviation in spending is actually higher in the flexible case. Without the undersupply incentive, the sales in the first period of both flexible and committed converge. Therefore, both settings have variable sales in the first period. In the second period, the variance of committed spending is mostly determined by the underlying demand uncertainty. In the flexile setting, the policy readjustment compounds the variance of the first period sales with the second period. This increases the variance in the spending distribution.

Figure 4



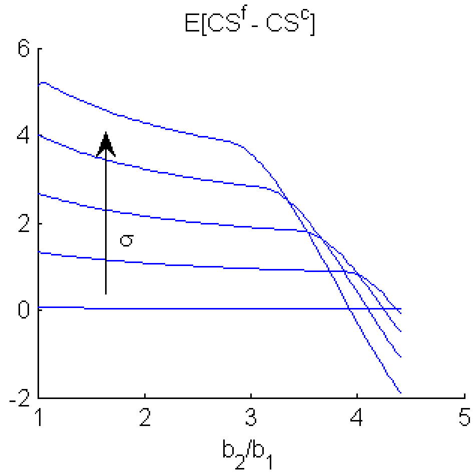
In Figure 4, we present the effect of demand correlation on the expected government spending. For $\alpha \geq 0$, the relationship of Corollary 1 is verified: $E[Spending^f - Spending^c] > 0$. Interestingly, when demand uncertainty, σ, is sufficiently high and the correlation is sufficiently negative, for instance $\alpha = -2$, the relationship can be inverted: $E[Spending^f - Spending^c] < 0$. This means that the *correlation effect* becomes dominant in Theorem 1. As seen in Theorem 1, the average spending for the committed setting does not depend on α. The flexible spending is what changes with α, as displayed in Figure 4.

Positive α means that low initial demand is followed by a lower average demand later. In the flexible setting, the government will overcompensate in subsidies to get back close to the adoption target. At the same time, subsidizing becomes increasingly expensive when arriving in the second period with low sales. When there is high early demand, the flexible government can reduce spending in the second period, but the benefits of high early demand are curbed by the limited supply.

With negative correlation, low initial demand is compensated by high demand later. High early demand leads to lower demand later. This effectively works as a natural hedge for the flexible government and can outweigh the other effects described in Theorem 1.

We next show computationally that the total expected consumer surplus can be larger or smaller in the flexible setting depending on the price sensitivity ratio. In Figure 5 is shown the ratio $\frac{E[CS^c]}{E[CS^f]}$ as a function of the ratio of price sensitivities b_2/b_1. More specifically, in this experiment, we fixed b_1 at the original estimated value and varied only b_2.

Figure 5



Observation 5: One can see that the total expected consumer surplus inequality can go either way depending on the value of b_2/b_1. When b_2/b_1 is not very large, the expected consumer surplus is higher in the flexible setting. However, for large values of b_2/b_1, the situation is reverted. This region where $E[CS^f - CS^c] < 0$, represents the regime where the benefit of higher subsidies in the flexible setting are dominated by the increased risk of early stock-outs. As shown in Theorem 4, the flexible setting does not always benefit consumers in terms of expected consumers surplus. We

further note the threshold value of b_2/b_1 that changes the consumer surplus preference is decreasing in the magnitude of the demand uncertainty.

5. Conclusions

Flexibility can be seen as an asset in a lot of operations management applications. When the government is designing consumer subsidies, policy flexibility can be a liability for the government. This result comes from the fact that industry is strategically responding to the policy design. Under a flexible policy, the firm will supply less in the early stage, relative to a committed policy. This is due to the fact that a low demand in the earlier period can be compensated by the government in the future, creating an undersupply incentive for the firm. This increases the total cost of the subsidy program.

As mentioned before, the results developed in this paper are not limited to settings with a monopolist supplier. We can obtain the same results if we assume symmetric firms compete to supply the product with a deterministic split of demand across suppliers. This means that the monopolist is not manipulating the government subsidy, otherwise the supplier competition would eliminate this manipulation. In fact, it is the government that is undermining the supply incentives of the firm (or firms) by readjusting policies over time. An interesting direction for future research would be to understand if a random demand splitting or asymmetric suppliers could alleviate the undersupply incentive of the flexible setting.

This result carries a potentially significant qualitative insight for policy makers. The constant readjustment of the subsidy policies can cause serious adverse effects in the production incentives. Governments should be careful about revising their policies and consider this unintended consequence of policy flexibility.

On the other hand, flexible policies obtain a lower variance in total sales. In other words, the flexible policy typically gets closer to the desired adoption level than the committed policy. We have also shown that a significant negative demand correlation across time periods creates an advantage for the flexible policy. Under negative correlation, the flexible spending might even be smaller than the committed spending. It is interesting to note that acquiring new demand information is not universally better for the flexible policy. In fact, only negative demand correlation provides a benefit to flexible policies in terms of average cost.

Finally, we note firms on average benefit from flexible subsidy policies, because of the reduced cost of undersupplying. Consumers may be better off or worse off with respect to policy commitment. Flexibility creates lower initial supply levels, which translates in higher stock-out risk. At the same time, it increases the average subsidy level. From a consumer's perspective, the trade-off between higher subsidy and higher stock-out probability will depend on the relative price elasticity of early customers and late customers.

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6. Appendix

6.1. Proof of Lemma 1

We first consider the committed setting. We denote by $h_2^c(x_2)$ the second period supplier's profit-to-go function and $H_2^c(x_2,u_2)$ as the second period supplier's objective function. Consider the supplier's problem at $t=2$, given by:

$$\begin{aligned} h_2^c(x_2) &= \max_{u_2 \geq 0} H_2^c(x_2,u_2) \\ &= \max_{u_2 \geq 0} p_2 E[\min(x_2+u_2, b_2r_2+\mu_2+\alpha w_1+w_2)] - c_2 u_2 \\ &\quad + p_3 E[\max(x_2+u_2-b_2r_2-\mu_2-\alpha w_1-w_2, 0)]. \end{aligned} \tag{10}$$

Let $\hat{u}_2$ be the solution of the first order condition of the problem above, which follows:

$$p_2 P(x_2+\hat{u}_2 \leq b_2r_2+\mu_2+\alpha w_1+w_2) - c_2 + p_3 P(x_2+\hat{u}_2-b_2r_2-\mu_2-\alpha w_1-w_2 \geq 0) = 0.$$

Which is equivalent to:

$$p_2(1-F_{w_2}(x_2+\hat{u}_2-b_2r_2-(\mu_2+\alpha w_1)) - c_2 + p_3 F_{w_2}(x_2+\hat{u}_2-b_2r_2-(\mu_2+\alpha w_1)) = 0.$$

The unique solution to the first order condition is given by:

$$\hat{u}_2 = b_2r_2 - x_2 + (\mu_2+\alpha w_1) + F_{w_2}^{-1}\left(\frac{p_2-c_2}{p_2-p_3}\right) = b_2r_2 - x_2 + (\mu_2+\alpha w_1) + k_2^c.$$

In addition, the second derivative of the objective function is non-positive:

$$\frac{d^2 H_2^c(x_2,u_2)}{du_2^2} = -p_2 f_{\epsilon_2}(x_2+u_2-b_2r_2) + p_3 f_{\epsilon_2}(x_2+u_2-b_2r_2) \leq 0.$$

Note here that $f_{\epsilon_2}(\cdot)$ is the pdf of ϵ_2, which is always positive. Since $p_2 > p_3$, the second order condition is satisfied and $\hat{u}_2$ is the maximizer of the unconstrained problem.

From Assumption 2, we have that $c_2 > p_3$. If this was not the case, the supplier could produce an infinite number of units during the second period at a cost below the salvage value, making infinite profits. Since $c_2 > p_3$, in the limit $u_2 \to \infty$, the objective function goes to: $H_2^c(x_2,u_2) \to -\infty$. From continuity of the objective function $H_2^c(x_2,u_2)$, there is a solution of the maximization problem above, which must be either at the boundary $u_2 = 0$ or satisfying the first order condition, i.e., $u_2 = \hat{u}_2$.

Since the objective value is finite at $u_2 = 0$ and $-\infty$ when $u_2 \to \infty$, the objective function $H_2^c(x_2,u_2)$ is non-increasing with respect to u_2 for any $u_2 \geq \hat{u}_2$. Therefore, the optimal second period ordering level in the committed setting is given by:

$$u_2^*(x_2,r_2) = \max(b_2r_2 - x_2 + k_2^c, 0).$$

At the first period, the manufacturer is solving the following problem in order to maximize the expected first period profit plus the profit-to-go of the second period:

$$\max_{u_1 \geq 0} \; p_1 E[\min(x_1+u_1, b_1 r_1+\epsilon_1)] - c_1 u_1 + E[h_2^c(x_2(x_1,u_1,r_1,\epsilon_1))]. \tag{11}$$

We define the following first period production quantity: $\hat{u}_1 = b_1 r_1 - x_1 + k_1^c + \mu_1$.

We will next show that this quantity satisfies the first order condition of the problem in (11). First, note that under this policy and Assumption 3, we obtain the no idling condition. If there is any left over inventory, it will be given by:

$$x_2 = x_1 + \hat{u}_1 - b_1 r_1 - \mu_1 - w_1 = k_1^c - w_1 \leq k_2^c + \mu_2 + \alpha w_1 \leq b_2 r_2 + k_2^c + \mu_2 + \alpha w_1.$$

The first inequality comes from Assumption 3 and the second from the non-negativity of the subsidy level. Therefore, the optimal second period ordering policy simplifies to $u_2^*(x_2, r_2, w_1) = b_2 r_2 - x_2 + k_2^c + \mu_2 + \alpha w_1$, which is non-negative. Under the optimal ordering policy, the profit-to-go is given by:

$$h_2^c(x_2) = p_2(b_2 r_2 + \mu_2 + \alpha w_1 + E[\min(k_2^c, w_2)] - c_2(b_2 r_2 - x_2 + \mu_2 + \alpha w_1 + k_2^c) + p_3 E[\max(k_2^c - w_2, 0)].$$

We next compute the derivative of the expected profit-to-go function:

$$\frac{dE[h_2^c]}{du_1} = E\left[\left(\frac{dh_2^c}{dx_2}\right)\left(\frac{dx_2}{du_1}\right)\right] = (c_2)\left(F_{w_1}(x_1 + u_1 - b_1 r_1 - \mu_1 - w_1)\right).$$

Therefore, the first order condition of equation (11) can be expressed as:

$$p_1(1 - F_{w_1}(x_1 + u_1 - b_1 r_1 - w_1)) - c_1 + c_2 F_{w_1}(x_1 + u_1 - b_1 r_1 - \mu_1) = 0.$$

Note that $\hat{u}_1$ is the unique solution to the expression above. Note also that the second order derivative is negative, guaranteeing optimality: $-p_1 f_{\epsilon_1}(x_1 + u_1 - b_1 r_1)) + c_2 f_{\epsilon_1}(x_1 + u_1 - b_1 r_1) < 0$. This follows from the facts that $p_1 > p_2 > c_2$ and the pdf $f_{\epsilon_1}(\cdot)$ is always positive. Therefore, the optimal solution is either at $u_1 = \hat{u}_1$ or at the boundary, i.e., $u_1 = 0$. From Assumption 3, we know that $\hat{u}_1 > 0$ and as a result: $u_1^*(x_1, r_1) = b_1 r_1 - x_1 + k_1^c + \mu_1$.

We next consider the government problem in the committed setting:

$$\begin{array}{ll} \min\limits_{r_1, r_2 \geq 0} & r_1 E[s_1(x_1, u_1^{c*}(x_1, r_1), r_1, \epsilon_1)] + r_2 E[s_2(x_2, u_2^{c*}(x_2, r_2), r_2, \epsilon_2)] \\ s.t. & E[s_1(x_1, u_1^{c*}(x_1, r_1), r_1, \epsilon_1)] + E[s_2(x_2, u_2^{c*}(x_2, r_2), r_2, \epsilon_2)] \geq \Gamma \\ \text{where:} & s_t(x_t, u_t, r_t, \epsilon_t) = \min(x_t + u_t, b_t r_t + \epsilon_t) \\ & x_{t+1} = x_t + u_t - s_t(x_t, u_t, r_t, \epsilon_t) \end{array} \tag{12}$$

Using the optimal production quantities, $u_1^{c*}(x_1, r_1)$ and $u_2^{c*}(x_2, r_2)$, derived above, we obtain the following expected sales levels: $E[s_t(x_t, u_t, r_t, \epsilon_t)] = b_t r_t + \mu_t + E[\min(k_t^c, w_t)] = b_t r_t + \mu_t + v_t^c$. As a result, the optimization problem reduces to:

$$\begin{array}{ll} \min\limits_{r_1, r_2 \geq 0} & r_1(b_1 r_1 + \mu_1 + v_1^c) + r_2(b_2 r_2 + \mu_2 + v_2^c) \\ s.t. & (b_1 r_1 + \mu_1 + v_1^c) + (b_2 r_2 + \mu_2 + v_2^c) \geq \Gamma \end{array} \tag{13}$$

The objective function is non-decreasing in both r_1 and r_2, and the expected sales is a continuous function. Therefore, the optimal solution must occur when the adoption constraint is exactly met. We can solve this by expressing r_1 as a function of r_2: $r_1 = (\Gamma - v_1^c - b_2 r_2 - v_2^c - \mu_1 - \mu_2)/b_1$. The problem becomes:

$$\min_{r_2 \geq 0} \frac{\Gamma - v_1^c - b_2 r_2 - v_2^c - \mu_1 - \mu_2}{b_1}(\Gamma - b_2 r_2 - v_2^c - \mu_2) + r_2(b_2 r_2 + \mu_2 + v_2^c). \tag{14}$$

Note that the objective function is convex in r_2. By taking the first order condition, we obtain:

$$r_2^{c*} = \frac{\Gamma}{b_1 + b_2} - \frac{(v_2^c + \mu_2)(b_1 + 2b_2)}{2b_2(b_1 + b_2)} - \frac{v_1^c + \mu_1}{2(b_1 + b_2)}.$$

The first period subsidy value follows from the target constraint and is given by:

$$r_1^{c*} = \frac{\Gamma}{b_1 + b_2} - \frac{(v_1^c + \mu_1)(2b_1 + b_2)}{2b_1(b_1 + b_2)} - \frac{v_2^c + \mu_2}{2(b_1 + b_2)}.$$

Similar to the optimal solution of the committed case, we next solve the flexible problem by starting from the second period. The derivation of the second period production level u_2 is the same as in the committed setting. In particular, the problem can be written as:

$$\begin{aligned} h_2^f(x_2, r_2) = \max_{u_2 \geq 0} \; & p_2 E[\min(x_2 + u_2, b_2 r_2 + \mu_2 + \alpha w_1 + w_2)] - c_2 u_2 \\ & + p_3 E[\max(x_2 + u_2 - b_2 r_2 - \mu_2 - \alpha w_1 - w_2, 0)]. \end{aligned} \tag{15}$$

The optimal ordering quantity can be expressed as: $u_2^*(x_2, r_2) = \max(b_2 r_2 - x_2 + k_2^f + \mu_2 + \alpha w_1, 0)$. The government optimization problem at the second period is given by:

$$\begin{aligned} g(s_1, x_2) = \min_{r_2} \; & r_2 E[s_2(x_2, u_2^*(x_2, r_2, w_1), r_2, w_1)] \\ s.t. \; & s_1 + E[s_2(x_2, u_2^{c*}(x_2, r_2, w_1), r_2, w_1)] \geq \Gamma. \end{aligned} \tag{16}$$

By using the optimal ordering quantity, we obtain:

$$E[s_2(x_2, u_2, r_2, \epsilon_2)] = b_2 r_2 + \mu_2 + E[\min(k_2^f, w_2)] = b_2 r_2 + \mu_2 + v_2^f.$$

One can see that both the objective function and the adoption constraint are non-decreasing with respect to r_2. Therefore, the optimal solution can be obtained when the adoption constraint is exactly met, that is:

$$r_2^{f*}(s_1, x_2) = \frac{\Gamma - s_1 - v_2^f - \mu_2 - \alpha w_1}{b_2}.$$

We next consider solving the supplier's problem for the first period. In order to find the optimal production quantity at the first period, we assume that the supplier knows the government' s response. This yields to the following problem:

$$\begin{aligned} \max_{u_1 \geq 0} \; & p_1 E[s_1(x_1, u_1, r_1, w_1)] - c_1 u_1 \\ & + E[h_2^f(x_2(x_1, u_1, r_1, w_1), r_2^*(s_1(x_1, u_1, r_1, w_1), x_2(x_1, u_1, r_1, w_1))]. \end{aligned} \tag{17}$$

As in the committed setting, we assume that the manufacturer does not idle in the second period. Note that we have: $h_2^f = p_2(b_2r_2 + \mu_2 + v_2^f + \alpha w_1) - c_2(b_2r_2 + \mu_2 + \alpha w_1 - x_2 + k_2^f) + p_3E[\max(k_2^f - w_2, 0)]$. Substituting the second period subsidy level, we obtain: $h_2^f = p_2(\Gamma - s_1) - c_2(\Gamma - s_1 - x_2) + p_3E[\max(k_2^f - w_2, 0)]$. Note also that $s_1 = \min(x_1 + u_1, b_1r_1 + \mu_1 + w_1)$ and $dE[h_2^f]/du_1 = -p_2(1 - F_{\epsilon_1}(x_1 + u_1 - b_1r_1)) + c_2$. The first order condition on problem (17) yields:

$$p_1(1 - F_{\epsilon_1}(x_1 + u_1 - b_1r_1)) - c_1 - p_2(1 - F_{\epsilon_1}(x_1 + u_1 - b_1r_1)) + c_2 = 0.$$

Equivalently: $(p_1 - p_2)F_{\epsilon_1}(x_1 + u_1 - b_1r_1)) = p_1 - c_1 - p_2 + c_2$. Note that the second derivative is negative, since $p_1 > p_2$, which implies that the objective function is concave. One see that the following first period production quantity uniquely satisfies the first order condition written above: $\hat{u}_1^f = b_1r_1 - x_1 + k_1^f + \mu_1$. Since from Assumption 3 we have $x_1 \le k_1^f + \mu_1$, we know that the optimal solution is positive and therefore $u_1^{f*} = \hat{u}_1^f$.

Under this policy, we have $s_1(x_1, \hat{u}_1, r_1, w_1) = b_1r_1 + \mu_1 + \min(k_1^f, w_1)$. In addition, $x_2 = x_1 + u_1 - s_1 = k_1^f - \min(k_1^f, w_1)$. Therefore, the second period subsidy level can be expressed as the first period subsidy as follows: $r_2^{f*}(s_1, x_2) = \dfrac{\Gamma - b_1r_1 - \mu_1 - \min(k_1^f, w_1) - \mu_2 - \alpha w_1 - v_2^f}{b_2}$.

The first period government problem is given by:

$$\min_{r_1} \; r_1E[s_1(x_1, u_1^*(r_1), r_1, w_1)] + E[g(s_1(x_1, u_1^*(r_1), r_1, w_1))]. \tag{18}$$

Note that from the solution of the second period problem, we have:

$$g(s_1) = \frac{\Gamma - s_1 - v_2^f}{b_2}(\Gamma - s_1) = \frac{\Gamma - (b_1r_1 + \min(k_1^f, \epsilon_1)) - v_2^f}{b_2}\left[\Gamma - (b_1r_1 + \min(k_1^f, \epsilon_1))\right].$$

The optimal subsidy level for the first period can then be obtained by solving the first order condition of problem (18). We further note that the second derivative is always positive, indicating that the function is convex. This solution is given by:

$$r_1^{f*} = \frac{\Gamma}{b_1 + b_2} - \frac{(v_1^f + \mu_1)(2b_1 + b_2)}{2b_1(b_1 + b_2)} - \frac{v_2^f + \mu_2}{2(b_1 + b_2)}.$$

□

6.2. Proof of Proposition 1

- From the definitions of the ordering quantiles in Table 1, we have: $k_1^c = F_{w_1}^{-1}\left(1 - \frac{c_1 - c_2}{p_1 - c_2}\right)$ and $k_1^f = F_{w_1}^{-1}\left(1 - \frac{c_1 - c_2}{p_1 - p_2}\right)$.

We assume w_1 to be continuously distributed with full support on $[A_1, B_1]$. Since the function $F_{w_1}^{-1}$ is increasing, we need only to show $1 - \frac{c_1 - c_2}{p_1 - c_2} > 1 - \frac{c_1 - c_2}{p_1 - p_2}$. This can be implied from $p_1 - c_2 > p_1 - p_2$, which is true from Assumption 2: $p_2 > c_2$. Therefore, $k_1^c > k_1^f$. For the second time period, the relationship is trivially true from the definition $k_2^c = k_2^f$.

Additionally, from the definition of the expected sales quantiles, we have: $v_1^c = E[\min(k_1^c, w_1)]$ and $v_1^f = E[\min(k_1^f, w_1)]$. Note that $\min(k_1^c, w_1) \geq \min(k_1^f, w_1)$ for any value of w_1. Since the distribution is fully supported in $[A_1, B_1]$ and we know that $A_1 < k_1^f < k_1^c < B_1$, so there will be some measurable part of the distribution where the inequality is strict: $\min(k_1^c, w_1) > \min(k_1^f, w_1)$. Therefore, we obtain $v_1^c > v_1^f$. In addition, recall that we have $v_2^c = v_2^f$.

- Using the optimal ordering policies and subsidy levels from Lemma 1, the expected sales are given by:

$$E[s_t] = E[\min\{x_t + u_t, b_t r_t + \epsilon_t\}] = b_t E[r_t^{j^*}] + \mu_t + E[\min\{k_t^j, w_t\}] = b_t E[r_t^{j^*}] + \mu_t + v_t^j,$$

where j can be either c or f for committed or flexible setting respectively. Note that the correlation effect is additive with zero mean, therefore not appearing in the expectation of sales. The first and second period expected sales level will be given by:

$$E[s_1^j] = \frac{b_1 \Gamma}{b_1 + b_2} + \frac{b_2(v_1^j + \mu_1)}{2(b_1 + b_2)} - \frac{b_1(v_2^j + \mu_2)}{2(b_1 + b_2)}$$

$$E[s_2^j] = \frac{b_2 \Gamma}{b_1 + b_2} + \frac{b_1(v_2^j + \mu_2)}{2(b_1 + b_2)} - \frac{b_2(v_1^j + \mu_1)}{2(b_1 + b_2)}$$

Note that the average sales maintain the same structure between the two settings $j \in \{c, f\}$. The only difference is the first expected sales quantile v_1^c and v_1^f. We can now calculate the difference of expected sales, where the terms without v_1^c and v_1^f will cancel each other therefore obtaining the desired result:

$$\begin{aligned} E[s_1^c - s_1^f] &= \tfrac{b_2}{2(b_1+b_2)}(v_1^c - v_1^f) > 0 \\ E[s_2^c - s_2^f] &= -\tfrac{b_2}{2(b_1+b_2)}(v_1^c - v_1^f) < 0 \end{aligned} \tag{19}$$

It is also easy to see that $E[s_1^j] + E[s_2^j] = \Gamma$, for both $j \in \{c, f\}$, which is a good sanity check for the solution. This is to be expected since the government uses this condition as a tight constraint in the optimization problem.

- By using Lemma 1, the expressions for u_1^c and u_1^f are given by:

$$u_1^c = b_1 r_1^c + k_1^c + \mu_1 - x_1$$

$$u_1^f = b_1 r_1^f + k_1^f + \mu_1 - x_1$$

Therefore, one can compute the difference:

$$u_1^c - u_1^f = b1(r_1^c - r_1^f) + k_1^c - k_1^f. \tag{20}$$

Next, we substitute the expressions for r_1^c and r_1^f from Lemma 1, given by:

$$r_1^c = \frac{\Gamma}{b_1+b_2} - \frac{(v_1^c+\mu_1)(2b_1+b_2)}{2b_1(b_1+b_2)} - \frac{v_2^c+\mu_2}{2(b_1+b_2)}$$
$$r_1^f = \frac{\Gamma}{b_1+b_2} - \frac{(v_1^f+\mu_1)(2b_1+b_2)}{2b_1(b_1+b_2)} - \frac{v_2^f+\mu_2}{2(b_1+b_2)}$$

So the difference is equal to:

$$r_1^c - r_1^f = (v_1^f - v_1^c)\frac{2b_1+b_2}{2b_1(b_1+b_2)}. \tag{21}$$

By replacing equation (21) in equation (20), we obtain:

$$u_1^c - u_1^f = k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1+b_2}{2(b_1+b_2)}. \tag{22}$$

Now, since $0 \le \frac{2b_1+b_2}{2(b_1+b_2)} \le 1$, it remains to show that $k_1^c - k_1^f \ge v_1^c - v_1^f$.

To show $k_1^c - k_1^f \ge v_1^c - v_1^f$, consider the following difference: $v_1^c - v_1^f = E[\min(k_1^c, w_1) - \min(k_1^f, w_1)]$. We must look at the each realization of the following random variable: $\min(k_1^c, w_1) - \min(k_1^f, w_1)$. We divide the analysis into cases depending on the realization of w_1.

—Case 1: $w_1 \ge k_1^c$. Since $k_1^c \ge k_1^f$, then $\min(k_1^c, w_1) - \min(k_1^f, w_1) = k_1^c - k_1^f \le k_1^c - k_1^f$.

—Case 2: $w_1 \le k_1^f$. Since $k_1^c \ge k_1^f$, then $\min(k_1^c, w_1) - \min(k_1^f, w_1) = w_1 - w_1 = 0 \le k_1^c - k_1^f$

—Case 3: $k_1^f \le w_1 \le k_1^c$. We have: $\min(k_1^c, w_1) - \min(k_1^f, w_1) = w_1 - k_1^f \le k_1^c - k_1^f$

Therefore, in each case: $\min(k_1^c, w_1) - \min(k_1^f, w_1) \le k_1^c - k_1^f$. By taking the expectation, we obtain: $v_1^c - v_1^f = E[\min(k_1^c, w_1) - \min(k_1^f, w_1)] \le k_1^c - k_1^f$. Therefore we conclude: $u_1^c \ge u_1^f$.

We next compare the expected production quantities for the second time period. Using Lemma 1, the expressions for u_2^c and u_2^f are given by:

$$u_2^c = b_2 r_2^c + k_2^c + \mu_2 + \alpha w_1 - x_2^c$$
$$u_2^f = b_2 r_2^f + k_2^f + \mu_2 + \alpha w_1 - x_2^f$$

Note that in the above expressions, the only random variables are: r_2^f, x_2^c, w_1 (whose mean is zero) and x_2^f, whereas the remaining terms are deterministic. Therefore, by taking the expectation, we obtain:

$$E[u_2^c] = b_2 r_2^c + k_2^c + \mu_2 - E[x_2^c]$$
$$E[u_2^f] = b_2 E[r_2^f] + k_2^f + \mu_2 - E[x_2^f]$$

The difference is then given by: $E[u_2^f] - E[u_2^c] = b_2\Big(E[r_2^f] - r_2^c\Big) + k_2^f - k_2^c + E[x_2^c] - E[x_2^f]$. Recall that $k_2^f = k_2^c$, and therefore $E[u_2^f] - E[u_2^c] = b_2\Big(E[r_2^f] - r_2^c\Big) + E[x_2^c] - E[x_2^f]$. From Proposition 1, we know: $E[r_2^f] \ge r_2^c$ and therefore, we need to show that $E[x_2^c] \ge E[x_2^f]$. We have:

$$E[x_2^c] = x_1 + u_1^c - E[s_1^c]$$
$$E[x_2^f] = x_1 + u_1^f - E[s_1^f]$$

So the difference is equal to: $E[x_2^c] - E[x_2^f] = u_1^c - u_1^f + E[s_1^f] - E[s_1^c]$. By replacing the expression from (22), we obtain:

$$E[x_2^c] - E[x_2^f] = k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1 + b_2}{2(b_1 + b_2)} - (v_1^c - v_1^f)\frac{b_2}{2(b_1 + b_2)}.$$

By canceling terms, we obtain: $E[x_2^c] - E[x_2^f] = k_1^c - k_1^f - (v_1^c - v_1^f)$. By using $k_1^c - k_1^f \geq v_1^c - v_1^f$, we conclude $E[x_2^c] \geq E[x_2^f]$, and therefore $E[u_2^f] \geq E[u_2^c]$.

Finally, we compare the total expected production quantities. The difference between the expected total production quantities is given by:

$$u_1^c - u_1^f - E\big[u_2^f - u_2^c\big] = k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1 + b_2}{2(b_1 + b_2)} - b_2\Big(E[r_2^f] - r_2^c\Big) - (k_1^c - k_1^f) - (v_1^f - v_1^c).$$

By canceling terms, we obtain:

$$u_1^c - u_1^f - E\big[u_2^f - u_2^c\big] = (v_1^c - v_1^f)\frac{b_2}{2(b_1 + b_2)} - b_2\Big(E[r_2^f] - r_2^c\Big).$$

From Lemma 1:

$$u_1^c - u_1^f - E\big[u_2^f - u_2^c\big] = (v_1^c - v_1^f)\frac{b_2}{2(b_1 + b_2)} - b_2\frac{v_1^c - v_1^f}{2(b_1 + b_2)} = 0.$$

Therefore, we conclude: $u_1^c + E[u_2^c] = u_1^f + E[u_2^f]$.

- From the definitions of the optimal subsidy levels in Lemma 1, we obtain the difference between the first period subsidy in the flexible and committed settings. This difference is given by:

$$r_1^{c*} - r_1^{f*} = -\frac{2b_1 + b_2}{2b_1(b_1 + b_2)}[v_1^c - v_1^f] < 0.$$

We know that $v_1^c > v_1^f$. Therefore, the subsidy level in the committed setting is smaller: $r_1^{c*} - r_1^{f*} < 0$. For the second period subsidy, we calculate the expected difference in subsidies:

$$r_2^{c*} - E[r_2^{f*}(s_1)] = -\frac{v_1^c}{2(b_1 + b_2)} + \frac{v_1^f}{2(b_1 + b_2)} < 0.$$

Which is also negative since $v_1^c > v_1^f$. $\square$

6.3. Proof of Theorem 1 and Corollary 1

Under a committed setting, the expected spending levels for each period is easily obtained since the subsidy levels are deterministic. $E[Spending^c] = E[s_1]r_1^{c*} + E[s_2]r_2^{c*}$. The expected sales will be given by $E[s_t] = \min\{x_t + u_t, b_t r_t + \epsilon_t\}$. Under the optimal ordering policy from Lemma 1 and considering that $E[w_1] = 0$, we obtain $E[s_t] = b_t r_t^{c*} + \mu_t + E[\min\{k_t^c, w_t\}] = b_t r_t^{c*} + \mu_t + v_t^c$. The first relationship is proven. Under a flexible setting, we obtain a $E[s_1]r_1^{f*} = b_1 r_1^{f*} + \mu_1 + v_1^f$ in a similar way for the first time period. For the second period, note that both subsidy and sales are random variables. Therefore, using $s_2 = b_f r_2^{f^*} + \mu_2 + \alpha w_1 + \min\{k_2^f, w_2\}$, we obtain the expectation of the product given by:

$$\begin{aligned}
E[s_2^f r_2^f] &= E[b_2(r_2^f)^2 + r_2^f(\mu_2 + \alpha w_1) + \min(k_2^f, w_2) r_2^f] \\
&= E[b_2(r_2^f)^2] + E[(\mu_2 + \alpha w_1)\frac{\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f}{b_2}] \\
&\quad + E[\min(k_2^f, w_2)\frac{\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f}{b_2}] \\
&= E[b_2(r_2^f)^2] + E[(\mu_2 + \alpha w_1)\frac{-s_1^f - \alpha w_1}{b_2}] + E[(\mu_2 + \alpha w_1)\frac{\Gamma - \mu_2 - v_2^f}{b_2}] + v_2^f\frac{\Gamma - \mu_2 - v_2^f}{b_2} \\
&\quad + E[\min(k_2^f, w_2)\frac{-s_1^f - \alpha w_1}{b_2}] \\
&= E[b_2(r_2^f)^2] - \frac{\mu_2}{b_2}E[s_1^f] - \frac{\alpha\mu_2}{b_2}E[(w_1)] - E[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}] + \frac{\mu_2(\Gamma - \mu_2 - v_2^f)}{b_2} \\
&\quad + \frac{\alpha(\Gamma - \mu_2 - v_2^f)}{b_2}E[w_1] + v_2^f\frac{\Gamma - \mu_2 - v_2^f}{b_2} - E[\min(k_2^f, w_2)\frac{s_1^f + \alpha w_1}{b_2}]
\end{aligned}$$

Since $E[w_1] = 0$ and w_1 and w_2 are independent, we obtain:

$$\begin{aligned}
E[s_2^f r_2^f] &= E[b_2(r_2^f)^2] - \frac{\mu_2}{b_2}E[s_1^f] - E[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}] \\
&\quad + \frac{(\mu_2 + v_2^f)(\Gamma - \mu_2 - v_2^f)}{b_2} - \frac{E[\min(k_2^f, w_2)s_1^f]}{b_2} - \alpha\frac{E[\min(k_2^f, w_2)]E[w_1]}{b_2}.
\end{aligned}$$

Using once more the independence assumption: $E[\min(k_2^f, w_2)s_1^f] = v_2^f E[s_1^f]$. Therefore:

$$\begin{aligned}
E[s_2^f r_2^f] &= E[b_2(r_2^f)^2] - \frac{\mu_2}{b_2}E[s_1^f] - E[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}] + \frac{(\mu_2 + v_2^f)(\Gamma - \mu_2 - v_2^f)}{b_2} - \frac{v_2^f E[s_1^f]}{b_2} \\
&= E[b_2(r_2^f)^2] - E[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}] + \frac{(\mu_2 + v_2^f)(\Gamma - \mu_2 - v_2^f - E[s_1^f])}{b_2}.
\end{aligned}$$

Recall that:

$$\frac{\Gamma - \mu_2 - v_2^f - E[s_1^f]}{b_2} = E[r_2^f].$$

Consequently, $E[s_2^f r_2^f] = E[b_2(r_2^f)^2] - E[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}] + (\mu_2 + v_2^f)E[r_2^f]$. Computing each term separately:

$$\begin{aligned}
E[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}] &= \frac{E[\alpha w_1(b_1 r_1^f + \mu_1 + min(k_1^f, w_1)) + (\alpha)^2(w_1)^2]}{b_2} \\
&= -\frac{(\alpha)^2 E[(w_1)^2]}{b_2} - \frac{\alpha E[w_1 min(k_1^f, w_1)]}{b_2}
\end{aligned}$$

The first term is given by:

$$\begin{aligned}
E[b_2(r_2^f)^2] &= b_2(Var(r_2^f) + (E[r_2^f])^2) = b_2 E[r_2^f]^2 + b_2\frac{Var(s_1^f + \alpha w_1)}{b_2} \\
&= b_2 E[r_2^f]^2 + \frac{Var(s_1^f)}{b_2} + \frac{(\alpha)^2 Var(w_1)}{b_2} + \frac{2Cov(s_1^f, \alpha w_1)}{b_2}
\end{aligned}$$

Note that: $Var(w_1) = E[w_1^2] - E[w_1]^2 = E[w_1^2]$. In addition, $\frac{2Cov(s_1^f, \alpha w_1)}{b_2} = \frac{2\alpha E[w_1 min(k_1^f, w_1)]}{b_2} - \frac{2\alpha E[min(k_1^f, w_1)]E[w_1]}{b_2} = \frac{2\alpha E[w_1 min(k_1^f, w_1)]}{b_2}$. Finally, we have: $Var(s_1^f) = Var(b_1 r_1^f + \mu_1 + min(k_1^f, w_1)) = Var(min(k_1^f, w_1))$. Therefore:

$$E[s_2^f r_2^f] = b_2 E[r_2^f]^2 + \frac{Var(min(k_1^f, w_1))}{b_2} + \frac{(\alpha)^2 E[(w_1)^2]}{b_2} + \frac{2\alpha E[w_1 min(k_1^f, w_1)]}{b_2}$$
$$- \frac{(\alpha)^2 E[(w_1)^2]}{b_2} - \frac{\alpha E[w_1 min(k_1^f, w_1)]}{b_2} + (\mu_2 + v_2^f) E[r_2^f].$$

As a result, we obtain:

$$E[s_2^f r_2^f] = (b_2 E[r_2^f] + v_2 + \mu_2) E[r_2^f] + \frac{Var(min(k_1^f, w_1))}{b_2} + \frac{\alpha E[w_1 min(k_1^f, w_1)]}{b_2}.$$

Since $v_2^f = v_2^c$, we will replace both of them by simply v_2. With some algebraic manipulations, we obtain:

$$\begin{aligned} E[Spending^f] = & \frac{Var(\min\{k_1^f, w_1\})}{b_2} + \frac{\alpha E[w_1 min(k_1^f, w_1)]}{b_2} \\ & + \frac{-(v_2+\mu_2)^2 b_1^2 - 4\Gamma b_2 (v_2+\mu_2) b_1 - 4\Gamma(v_1^f+\mu_1) b_2 b_1 + 4 b_1 b_2 \Gamma^2}{4 b_1 b_2 (b_1+b_2)} \\ & + \frac{2(v_1^f+\mu_1) b_2 (v_2+\mu_2) b_1 - (v_1^f+\mu_1)^2 b_2^2}{4 b_1 b_2 (b_1+b_2)} \end{aligned}$$

Similarly, for the committed setting we obtain:

$$\begin{aligned} E[Spending^c] = & \frac{-(v_2+\mu_2)^2 b_1^2 - 4\Gamma b_2 (v_2+\mu_2) b_1 - 4\Gamma(v_1^c+\mu_1) b_2 b_1 + 4 b_1 b_2 \Gamma^2}{4 b_1 b_2 (b_1+b_2)} \\ & + \frac{2(v_1^c+\mu_1) b_2 (v_2+\mu_2) b_1 - (v_1^c+\mu_1)^2 b_2^2}{4 b_1 b_2 (b_1+b_2)} \end{aligned}$$

When calculating the difference in spending, many terms cancel each other, leaving the following relation:

$$\begin{aligned} E[Spending^f] - E[Spending^c] = & \frac{Var(\min\{k_1^f, w_1\})}{b_2} + \frac{\alpha E[w_1 min(k_1^f, w_1)]}{b_2} + \\ & + \frac{1}{4 b_1 (b_1+b_2)} \left[2 b_1 (v_1^c - v_1^f)(2\Gamma - v_2^c - \mu_2) + b_2 (v_1^c+\mu_1)^2 - b_2 (v_1^f+\mu_1)^2 \right]. \end{aligned}$$

Note that $Var(\min\{k_1^f, w_1\}) > 0$. We also know that from Proposition 1: $v_1^c + \mu_1 > v_1^f + \mu_1 > 0$. Therefore, we get both $2b_1(v_1^c - v_1^f) > 0$ and $b_2(v_1^c+\mu_1)^2 - b_2(v_1^f+\mu_1)^2 > 0$. The remaining middle term $(2\Gamma - v_2^c - \mu_2)$ is also positive from the assumption that the target is large enough that the subsidy solution is non-trivial: $\Gamma > E[\epsilon_2]$, which is itself larger than the expected sales quantile: $\Gamma > E[\epsilon_2] > E[\min(k_2^c, \epsilon_2)] > E[\min(k_2^c, w_2)] = v_2^c$. Therefore: $E[Spending^f] > E[Spending^c]$. $\square$

6.4. Proof of Theorem 2

The difference of the variance of sales is given by:

$$Var(s^c) - Var(s^f) = E[(s^c)^2 - (s^f)^2] - E[s^c]^2 + E[s^f]^2.$$

We know that the expected sales is the same in both setting and equal to the target level: $E[s^c]^2 = E[s^f]^2 = \Gamma^2$. Then, we obtain: $Var(s^c) - Var(s^f) = E[(s^c)^2 - (s^f)^2]$. We now replace the total sales s^c and s^f by the sum at each time period: $s^c = s_1^c + s_2^c$ and $s^f = s_1^f + s_2^f$, so that we have:

$$Var(s^c) - Var(s^f) = E[(s_1^c)^2] + E[(s_2^c)^2] - E[(s_1^f)^2] - E[(s_2^f)^2] + 2E[s_1^c s_2^c - s_1^f s_2^f].$$

By definition, the sales of period 1 under the committed setting are given by:

$$s_1^c = \min(x_1 + u_1^c, b_1 r_1^c + \epsilon_1) = \min(b_1 r_1^c + k_1^c + \mu_1, b_1 r_1^c + \mu_1 + w_1) = b_1 r_1^c + \mu_1 + \min(k_1^c, w_1).$$

Then, the second moment can be written as follows:

$$E[(s_1^c)^2] = b_1^2 (r_1^c)^2 + \mu_1^2 + 2b_1 r_1^c(\mu_1 + v_1^c) + 2\mu_1 v_1^c + E[\min(k_1^c, w_1)^2].$$

Similarly, we have: $E[(s_1^f)^2] = b_1^2 (r_1^f)^2 + +\mu_1^2 + 2b_1 r_1^f(\mu_1 + v_1^f) + 2\mu_1 v_1^f + E[\min(k_1^f, w_1)^2]$. For s_2^c, the correlation appears.

$$\begin{aligned} s_2^c = \min(x_2^c + u_2^c, b_2 r_2^c + \epsilon_2) &= \min(b_2 r_2^c + k_2^c + \mu_2 + \alpha w_1, b_2 r_2^c + \mu_2 + \alpha w_1 + w_2) \\ &= b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2). \end{aligned}$$

Therefore: $E[(s_2^c)^2] = b_2^2 (r_2^c)^2 + \mu_2^2 + 2b_2 r_2^c(\mu_2 + v_2^c) + 2\mu_2 v_2^c + E[\min(k_2^c, w_2)^2] + \alpha^2 E[w_1^2]$.

However, r_2^f is a random variable, therefore the expectation $E[(s_2^f)^2]$ is calculated differently.

$$\begin{aligned} s_2^f &= b_2 r_2^f + \mu_2 + \alpha w_1 + \min(k_2^f, w_2) \\ &= (\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f) + \mu_2 + \alpha w_1 + \min(k_2^f, w_2) \\ &= \Gamma - s_1^f - v_2^f + \min(k_2^f, w_2) \end{aligned}$$

Then:

$$(s_2^f)^2 = (\Gamma - v_2^f)^2 + (s_1^f)^2 + (\min(k_2^f, w_2))^2 - 2s_1^f(\Gamma - v_2^f) - 2s_1^f \min(k_2^f, w_2) + 2(\Gamma - v_2^f)\min(k_2^f, w_2).$$

Considering that s_1^f and $\min(k_2^f, w_2)$ are independent, we obtain:

$$\begin{aligned} &E[(s_2^f)^2] = \\ &(\Gamma - v_2^f)^2 + E[(s_1^f)^2] + E[(\min(k_2^f, w_2))^2] - 2E[s_1^f](\Gamma - v_2^f) - 2(b_1 r_1^f + \mu_1 + v_1^f)v_2^f + 2(\Gamma - v_2^f)v_2^f. \end{aligned}$$

We next look at the product $s_1^c s_2^c$. We know that:

$$s_1^c s_2^c = \Big(b_1 r_1^c + \mu_1 + \min(k_1^c, w_1)\Big)\Big(b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)\Big).$$

By expanding the expression, we obtain:

$$s_1^c s_2^c = b_1 r_1^c (b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)) + \mu_1 (b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)) \\ + \min(k_1^c, w_1)(b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)).$$

Since w_1 and w_2 are assumed to be independent, and $E[w_1] = 0$ we have:

$$E[s_1^c s_2^c] = \\ b_1 b_2 r_1^c r_2^c + b_1 r_1^c \mu_2 + b_1 r_1^c v_2^c + \mu_1 b_2 r_2^c + \mu_1 \mu_2 + \mu_1 v_2^c + b_2 r_2^c v_1^c + \mu_2 v_1^c + \alpha E[w_1 min(k_1^c, w_1)] + v_1^c v_2^c.$$

Similarly, we have: $s_1^f s_2^f = s_1^f \Big(b_2 r_2^f + \mu_2 + \alpha w_1 + \min(k_2^f, w_2) \Big)$. By replacing the expression for r_2^f, we obtain:

$$\begin{aligned} s_1^f s_2^f &= s_1^f \Big(\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f + \mu_2 + \alpha w_1 + \min(k_2^f, w_2) \Big) \\ &= s_1^f \Big(\Gamma - s_1^f - v_2^f + \min(k_2^f, w_2) \Big) \\ &= -(s_1^f)^2 + s_1^f (\Gamma - v_2^f) + s_1^f \min(k_2^f, w_2) \end{aligned}$$

By again using the independence assumption: $E[\min(k_1^f, w_1)\min(k_2^f, w_2)] = v_1^f v_2^f$, we obtain:

$$E[s_1^f s_2^f] = -E[(s_1^f)^2] + (\Gamma - v_2^f)(b_1 r_1^f + \mu_1 + v_1^f) + b_1 r_1^f v_2^f \mu_1 v_2^f + v_1^f v_2^f.$$

By simplifying the above expression: $E[s_1^f s_2^f] = \Gamma(b_1 r_1^f + \mu_1 + v_1^f) - E[(s_1^f)^2]$. We now substitute all the previous expressions in the difference of the variances:

$$\begin{aligned} Var(s^c) \; -&Var(s^f) = \\ & b_1^2 (r_1^c)^2 \mu_1^2 + E[\min(k_1^c, w_1)^2] + 2 b_1 r_1^c (\mu_1 + v_1^c) + 2\mu_1 v_1^c + b_2^2 (r_2^c)^2 \\ & + 2 b_2 r_2^c (\mu_2 + v_2^c) + \mu_2^2 + E[\min(k_2^c, w_2)^2] + \alpha^2 E[w_1^2] - E[(s_1^f)^2] \\ & - \Big((\Gamma - v_2^f)^2 + E[(s_1^f)^2] + E[(min(k_2^f, w_2)^2] - 2E[s_1^f](\Gamma - v_2^f) - 2(b_1 r_1^f + \mu_1 + v_1^f) v_2^f \\ & + 2(\Gamma - v_2^f) v_2^f \Big) + 2 \Big(b_1 b_2 r_1^c r_2^c + b_1 r_1^c \mu_2 + b_1 r_1^c v_2^c + \mu_1 b_2 r_2^c + \mu_1 \mu_2 + \mu_1 v_2^c + b_2 r_2^c v_1^c + \mu_2 v_1^c \\ & + \alpha E[w_1 min(k_1^c, w_1)] + v_1^c v_2^c \Big) - 2 \Big(\Gamma (b_1 r_1^f + \mu_1 + v_1^f) - E[(s_1^f)^2] \Big) \end{aligned}$$

By merging terms:

$$\begin{aligned} Var(s^c) \; -&Var(s^f) = \\ & (b_1 r_1^c + b_2 r_2^c)^2 + 2(\mu_1 + v_1^c + \mu_2 + v_2^c)(b_1 r_1^c + b_2 r_2^c) + 2(v_1^c + \mu_1)(v_2^c + \mu_2) + (\mu_2 + v_2^c)^2 \\ & - \Gamma^2 + \mu_1^2 + 2\mu_1 v_1^c + E[\min(k_1^c, w_1)^2] + \alpha^2 E[w_1^2] + 2\alpha E[w_1 min(k_1^c, w_1)] \end{aligned}$$

Note that: $(b_1 r_1^c + \mu_1 + v_1^c) + (b_2 r_2^c + \mu_2 + v_2^c) = E[s_1^c] + E[s_2^c] = \Gamma$. Therefore, we obtain:

$$\begin{aligned} Var(s^c) - Var(s^f) &= \Gamma^2 - (v_1^c)^2 - \Gamma^2 + Var(\alpha w_1) + E[\min(k_1^c, w_1)^2] + 2\alpha E[w_1 min(k_1^c, w_1)] \\ &= Var(\alpha w_1) + (E[\min(k_1^c, w_1)^2] - (v_1^c)^2) + 2Cov(\alpha w_1, min(k_1^c, w_1)) \\ &= Var(\alpha w_1) + Var(\min(k_1^c, w_1)) + 2Cov(\alpha w_1, min(k_1^c, w_1)) \\ &= Var(\alpha w_1 + \min(k_1^c, w_1)) \geq 0 \end{aligned}$$

Therefore, $Var(s^c) \geq Var(s^f)$ for any value of α. $\square$

6.5. Proof of Theorem 3

The expected total profits in both settings are given by:

$$E[\pi^c] = p_1 E[s_1^c] + p_2 E[s_2^c] - c_1 u_1^c - c_2 E[u_2^c]$$
$$E[\pi^f] = p_1 E[s_1^f] + p_2 E[s_2^f] - c_1 u_1^f - c_2 E[u_2^f]$$

By taking the difference, we obtain:

$$E[\pi^f - \pi^c] = p_1 E[s_1^f - s_1^c] + p_2 E[s_2^f - s_2^c] - c_1(u_1^f - u_1^c) - c_2 E[u_2^f - u_2^c].$$

By replacing the expressions for the sales and the production quantities, we obtain:

$$E[\pi^f - \pi^c] = (p_1 - p_2)(v_1^f - v_1^c)\frac{b_2}{2(b_1+b_2)} + (c_1 - c_2)\Big[k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1+b_2}{2(b_1+b_2)}\Big].$$

However, we know from the assumption on the profit margins that: $0 \le c_1 - c_2 \le p_1 - p_2$ and therefore:

$$E[\pi^f - \pi^c] \ge (c_1 - c_2)\Big[(v_1^f - v_1^c)\frac{b_2}{2(b_1+b_2)} + k_1^c - k_1^f + (v_1^f - v_1^c)\frac{2b_1+b_2}{2(b_1+b_2)}\Big].$$

By simplifying the above expression, we have: $E[\pi^f - \pi^c] \ge (c_1 - c_2)(v_1^f - v_1^c + k_1^c - k_1^f)$. From Lemma 1, we know that: $v_1^f - v_1^c + k_1^c - k_1^f \ge 0$ and we also have by assumption: $(c_1 - c_2) \ge 0$, so that one can conclude: $E[\pi^f] \ge E[\pi^c]$. $\square$

6.6. Proof of Theorem 4

1. We have the following expressions for CS_2^f:

$$CS_2^f = \frac{(b_2 r_2^f + \epsilon_2)\min(u_2^f + x_2^f, b_2 r_2^f + \epsilon_2)}{2b_2}.$$

We know that $u_2^f + x_2^f = b_2 r_2^f + k_2^f + \mu_2 + \alpha w_1$ and therefore:

$$CS_2^f = \frac{(b_2 r_2^f + \epsilon_2)(b_2 r_2^f + \min(k_2^f + \mu_2 + \alpha w_1, \epsilon_2))}{2b_2}.$$

Similarly, we obtain:

$$CS_2^c = \frac{(b_2 r_2^c + \epsilon_2)(b_2 r_2^c + \min(k_2^c + \mu_2 + \alpha w_1, \epsilon_2))}{2b_2}.$$

By taking the expectation and computing the difference:

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] &= \frac{E\Big[b_2^2 (r_2^f)^2 - b_2^2 (r_2^c)^2 + b_2 \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)(r_2^f - r_2^c) + b_2 \epsilon_2 (r_2^f - r_2^c)\Big]}{2b_2} \\ &= \frac{b_2\Big(E[(r_2^f)^2] - (r_2^c)^2\Big)}{2} + \frac{E\Big[(r_2^f - r_2^c)(\epsilon_2 + \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2))\Big]}{2} \end{aligned}$$

Here, we used the fact that $k_2^f = k_2^c = k_2$. By using the facts that $\epsilon_2 \geq \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)$ and $E[(r_2^f)^2] = Var(r_2^f) + E[(r_2^f)]^2$, we obtain:

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] &\geq \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2 Var(r_2^f)}{2} + E[(r_2^f - r_2^c)(\min(k_2 + \mu_2 + \alpha w_1, \epsilon_2))] \\ &= \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2 Var(r_2^f)}{2} + E[r_2^f \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)] - r_2^c(v_2^c + \mu_2) \end{aligned}$$

We know that $r_2^f = \dfrac{q - s_1^f - \alpha w_1 - v_2^f - \mu_2}{b_2}$ and therefore:

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] \geq \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2 Var(r_2^f)}{2} + \frac{E[(q - v_2^f - \mu_2)\min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)]}{b_2} \\ - \frac{E[(s_1^f + \alpha w_1)\min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)]}{b_2} - r_2^c(v_2^c + \mu_2). \end{aligned}$$

By using the facts $v_2^f = v_2^c = v_2$ and $s_1^f = b_1 r_1^f + \mu_1 + \min(k_1^f, w_1)$, we obtain:

$$\begin{aligned} & E[CS_2^f] - E[CS_2^c] \geq \\ & \quad \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2 Var(r_2^f)}{2} + \frac{(v_2 + \mu_2)(q - v_2 - \mu_2 - b_2 r_2^c)}{b_2} \\ & \quad - \frac{E[b_1 r_1^f \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2) + \alpha w_1 \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2) + \mu_1 \min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)]}{b_2} \\ & \quad + \frac{E[\min(k_1^f, w_1)\min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)]}{b_2}. \end{aligned}$$

But: $q - v_2 - \mu_2 - b_2 r_2^c = q - E[s_2^c] = E[s_1^c]$ and therefore:

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] \geq \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2 Var(r_2^f)}{2} + \frac{(v_2 + \mu_2)E[s_1^c]}{b_2} - \frac{(b_1 r_1^f + v_1^f + \mu_1)(v_2 + \mu_2)}{b_2} \\ - \frac{\alpha^2 E[w_1^2]}{b_2} - \frac{\alpha E[w_1 min(k_1, w_1)]}{b_2}. \end{aligned}$$

Now, since $b_1 r_1^f + \mu_1 + v_1^f = E[s_1^f]$, we obtain:

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] \geq \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2 Var(r_2^f)}{2} + \frac{(v_2 + \mu_2)(E[s_1^c] - E[s_1^f])}{b_2} \\ - \frac{\alpha^2 E[w_1^2]}{b_2} - \frac{\alpha E[w_1 min(k_1, w_1)]}{b_2} \end{aligned}$$

We have:

$$\begin{aligned} Var(r_2^f) &= Var(\frac{s_1^f + \alpha w_1}{b_2}) \\ Var(r_2^f) &= \frac{Var(s_1^f)}{b_2^2} + \frac{\alpha^2 Var(w_1)}{b_2^2} + \frac{2\alpha Cov(s_1^f, w_1)}{b_2^2} \\ Var(r_2^f) &= \frac{Var(min(k_1^f, w_1))}{b_2^2} + \frac{\alpha^2 E[w_1^2]}{b_2^2} + \frac{2\alpha Cov(s_1^f, w_1)}{b_2^2} \end{aligned}$$

We note that: $Cov(s_1^f, w_1) = E[w_1 min(k_1, w_1)]$. Therefore:

$$\frac{b_2 Var(r_2^f)}{2} = \frac{Var(min(k_1^f, w_1))}{2b_2} + \frac{\alpha^2 E[w_1^2]}{2b_2} + \frac{\alpha E[w_1 min(k_1, w_1)]}{b_2}.$$

Thus:

$$E[CS_2^f] - E[CS_2^c] \geq \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{Var(min(k_1^f, w_1))}{2b_2} + \frac{(v_2 + \mu_2)(E[s_1^c] - E[s_1^f])}{b_2} - \frac{\alpha^2 E[w_1^2]}{2b_2}.$$

2. We have the following expression for CS_1^f:

$$CS_1^f = \frac{(b_1 r_1^f + \epsilon_1)\min(u_1^f + x_1, b_1 r_1^f + \epsilon_1)}{2b_1}.$$

We know that $u_1^f + x_1 = b_1 r_1^f + k_1^f + \mu_1$ and $\epsilon_1 = \mu_1 + w_1$. Therefore:

$$CS_1^f = \frac{(b_1 r_1^f + \mu_1 + w_1)(b_1 r_1^f + \mu_1 + \min(k_1^f, w_1))}{2b_1}.$$

Similarly, for the committed setting we obtain:

$$CS_1^c = \frac{(b_1 r_1^c + \mu_1 + w_1)(b_1 r_1^c + \mu_1 + \min(k_1^c, w_1))}{2b_1}.$$

By taking the expectation and computing the difference:

$$\begin{aligned} E[CS_1^f] - E[CS_1^c] &= \frac{E[(b_1 r_1^f + \mu_1 + w_1)(b_1(r_1^f) + \mu_1) - (b_1 r_1^c + \mu_1 + w_1)(b_1 r_1^c + \mu_1)]}{2b_1} \\ &+ \frac{E[(b_1 r_1^f + \mu_1 + w_1)\min(k_1^f, w_1) - (b_1 r_1^c + \mu_1 + w_1)\min(k_1^c, w_1)]}{2b_1} \\ &= \frac{E[(b_1 r_1^f + \mu_1)^2 - (b_1(r_1^c) + \mu_1)^2 + w_1(b_1 r_1^f - b_1 r_1^c) + (b_1 r_1^f + \mu_1)\min(k_1^f, w_1)]}{2b_1} \\ &- \frac{E[(b_1 r_1^c + \mu_1)\min(k_1^c, w_1) + w_1(\min(k_1^f, w_1) - \min(k_1^c, w_1))]}{2b_1} \end{aligned}$$

Since $E[w_1] = 0$, we obtain:

$$\begin{aligned} E[CS_1^f] - E[CS_1^c] &= \frac{(b_1 r_1^f + \mu_1)^2 - (b_1 r_1^c + \mu_1)^2 + (b_1 r_1^f + \mu_1)v_1^f - (b_1 r_1^c + \mu_1)v_1^c}{2b_1} \\ &+ \frac{E[w_1(\min(k_1^f, w_1) - \min(k_1^c, w_1))]}{2b_1}. \end{aligned}$$

Note that one can write:

$$r_1^f = r_1^c + \frac{(2b_1 + b_2)(v_1^c - v_1^f)}{2b_1(b_1 + b_2)}.$$

Therefore, we obtain:

$$
\begin{aligned}
&E[CS_1^f]-E[CS_1^c]=\\
&=\frac{(b_1r_1^c+\mu_1+\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)})^2-(b_1r_1^c+\mu_1)^2+(b_1r_1^c+\mu_1+\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)})v_1^f-(b_1r_1^c+\mu_1)v_1^c}{2b_1}\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&=\frac{(b_1r_1^c+\mu_1)^2-(b_1r_1^c+\mu_1)^2+(\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)})^2+(b_1r_1^c+\mu_1)\frac{(2b_1+b_2)(v_1^c-v_1^f)}{(b_1+b_2)}}{2b_1}\\
&\quad+\frac{(b_1r_1^c+\mu_1)(v_1^f-v_1^c)+v_1^f\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)}+E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&=\frac{(\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)})^2+(b_1r_1^c+\mu_1)\frac{(2b_1+b_2)(v_1^c-v_1^f)}{(b_1+b_2)}+(b_1r_1^c+\mu_1-v_1^f\frac{(2b_1+b_2)}{2(b_1+b_2)})(v_1^f-v_1^c)}{2b_1}+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&=\frac{(\frac{(2b_1+b_2)}{2(b_1+b_2)})^2(v_1^c-v_1^f)^2+(v_1^c-v_1^f)((b_1r_1^c+\mu_1)\frac{b_1}{b_1+b_2}+v_1^f\frac{(2b_1+b_2)}{2(b_1+b_2)})}{2b_1}+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}
\end{aligned}
$$

We next re-write the above expression as a function of the ratio of the price sensitivities $\theta=\frac{b_2}{b_1}$:

$$
\begin{aligned}
&E[CS_1^f]-E[CS_1^c]=\\
&=\frac{v_1^c-v_1^f}{2b_1}[(v_1^c-v_1^f)\frac{(2+\theta)^2}{(2(1+\theta))^2}+\frac{b_1r_1^c(\theta)+\mu_1}{1+\theta}+v_1^f\frac{2+\theta}{2(1+\theta)}]+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&=\frac{v_1^c-v_1^f}{2b_1(1+\theta)}[(v_1^c-v_1^f)\frac{(2+\theta)^2}{4(1+\theta)}+b_1r_1^c(\theta)+\mu_1+v_1^f\frac{2+\theta}{2}]+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&=\frac{v_1^c-v_1^f}{2b_1(1+\theta)}[\frac{(1+\theta/2)^2}{1+\theta}(v_1^c-v_1^f)+b_1r_1^c(\theta)+\mu_1+v_1^f(1+\frac{\theta}{2})]+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}
\end{aligned}
$$

We next show that the term $E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]$ is always non-positive. Since $k_1^f\le k_1^c$, we have:

$$
\begin{aligned}
w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))&=0,\ \text{when } w_1\le k_1^f\\
w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))=w_1(k_1^f-k_1^c)&\le 0,\ \text{when } w_1\ge k_1^c\\
w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))\le w_1(k_1^f-w_1)&\le 0,\ \text{when } k_1^f\le w_1\le k_1^c
\end{aligned}
$$

Therefore, $E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]\le 0$.

We next study the limits of $E[CS_1^f]-E[CS_1^c]$ when θ goes to zero and when θ goes to infinity. Without loss of generality, we assume that b_1 is a given constant and that b_2 is varying. Note that:

$$b_1r_1^c(\theta)=\frac{\Gamma-\frac{(v_2+\mu_2)}{2}-(v_1^c+\mu_1)(1+\frac{\theta}{2})}{1+\theta}.$$

By taking the limit, we obtain:

$$\lim_{\theta\to+\infty}b_1r_1^c(\theta)=-\frac{(v_1^c+\mu_1)}{2}.$$

As a result:

$$\lim_{\theta\to+\infty}E[CS_1^f]-E[CS_1^c]=\frac{(v_1^c-v_1^f)(v_1^c+v_1^f)+4E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{8b_1}\le 0.$$

We next study the limit when $\theta \to 0$:

$$\lim_{\theta\to 0} E[CS_1^f] - E[CS_1^c] = \frac{1}{2b_1} E[(\min(k_1^c, w_1) - \min(k_1^f, w_1))(\Gamma - \frac{v_2+\mu_2}{2} - w_1)].$$

We know that $\Gamma \geq 2w_1$. In addition, from Assumption 3.3, $\Gamma \geq 2(v_2+\mu_2)$. Therefore, we obtain: $\Gamma - \frac{v_2+\mu_2}{2} - w_1 \geq 0$ and therefore:

$$\lim_{\theta\to 0} E[CS_1^f] - E[CS_1^c] \geq 0.$$

Consequently, we have shown that when the ratio of price sensitivities is approaching zero, the consumers are better off in the flexible setting whereas when this ratio is approaching infinity, the consumers are better off in the committed setting, in terms of expected consumer surplus. In order to conclude the proof, we next show that the difference in expected consumer surplus, $E[CS_1^f] - E[CS_1^c]$ is non-increasing with respect to θ, i.e.,

$$\frac{\partial(E[CS_1^f] - E[CS_1^c])}{\partial\theta} \leq 0.$$

Recall that we have:

$$\begin{aligned} E[CS_1^f] - E[CS_1^c] = &\frac{(v_1^c - v_1^f)}{2b_1}\frac{1}{1+\theta}(\frac{(1+\theta/2)^2}{1+\theta}(v_1^c - v_1^f) + b_1 r_1^c(\theta) + \mu_1 + v_1^f(1+\frac{\theta}{2}) \\ &+ \frac{E[w_1(\min(k_1^f, w_1) - \min(k_1^c, w_1))]}{2b_1}. \end{aligned}$$

Note that the second term does not depend on θ. The derivative is given by:

$$\begin{aligned} \frac{\partial(E[CS_1^f] - E[CS_1^c])}{\partial\theta} = &\frac{v_1^c - v_1^f}{2b_1}\Big[\frac{(1+\theta)(\frac{v_1^f}{2} + (v_1^c - v_1^f)\frac{\theta(1+\frac{\theta}{2})}{2(1+\theta)^2} + b_1\frac{\partial r_1^c}{\partial\theta}}{(1+\theta)^2} \\ &- \frac{(v_1^c - v_1^f)\frac{(1+\frac{\theta}{2})^2}{1+\theta} + v_1^f(1+\frac{\theta}{2}) + \mu_1 + b_1 r_1^c(\theta)}{(1+\theta)^2}\Big]. \end{aligned}$$

By rearranging and simplifying, we obtain:

$$\begin{aligned} \frac{\partial(E[CS_1^f] - E[CS_1^c])}{\partial\theta} = &\frac{v_1^c - v_1^f}{2b_1(1+\theta)^2}[(v_1^c - v_1^f)\Big[\frac{\theta(1+\frac{\theta}{2})}{2(1+\theta)} - \frac{(1+\frac{\theta}{2})^2}{1+\theta}] + (1+\theta)b_1\frac{\partial r_1^c}{\partial\theta} - b_1 r_1^c(\theta) - \mu_1 \\ &+ v_1^f(\frac{1+\theta}{2} - \frac{2+\theta}{2})\Big]. \end{aligned}$$

Note that we have:

$$\begin{aligned} b_1 r_1^c(\theta) &= \frac{\Gamma - \frac{(v_2+\mu_2)}{2} - (v_1^c + \mu_1)(1+\frac{\theta}{2})}{1+\theta} \\ \frac{b_1\partial r_1^c}{\partial\theta} &= \frac{-\Gamma + \frac{v_2+\mu_2}{2} + \frac{v_1^c+\mu_1}{2}}{(1+\theta)^2} \end{aligned}$$

Therefore:

$$\begin{aligned}
\frac{\partial(E[CS_1^f]-E[CS_1^c])}{\partial\theta} &= \frac{v_1^c-v_1^f}{2b_1(1+\theta)^2}\left[-\mu_1-\frac{v_1^f}{2}-(v_1^c-v_1^f)\frac{1+\frac{\theta}{2}}{1+\theta}+\frac{-2\Gamma+(v_2+\mu_2)+(v_1^c+\mu_1)(\frac{3+\theta}{2})}{1+\theta}\right]\\
&= \frac{v_1^c-v_1^f}{2b_1(1+\theta)^2}\left[\mu_1\frac{(\frac{3+\theta}{2}-(1+\theta))}{1+\theta}+v_1^f(\frac{1}{2}+\frac{1}{2(1+\theta)}-\frac{1}{2})+v_1^c\frac{\frac{3+\theta}{2}-1-\frac{\theta}{2}}{1+\theta}+\frac{-2\Gamma+(v_2+\mu_2)}{1+\theta}\right]\\
&= \frac{v_1^c-v_1^f}{2b_1(1+\theta)^3}\left[\mu_1\frac{1-\theta}{2}+\frac{v_1^f}{2}+\frac{v_1^c}{2}+(-2\Gamma+(v_2+\mu_2))\right]\\
&= \frac{v_1^c-v_1^f}{2b_1(1+\theta)^3}\left[\frac{v_1^f}{2}-\frac{\theta\mu_1}{2}+(-2\Gamma+(v_2+\mu_2)+\frac{\mu_1+v_1^c}{2})\right]
\end{aligned}$$

Note that we have the following inequalities:

$$v_1^c - v_1^f \geq 0; \quad b_1 \geq 0; \quad v_1^f \leq 0; \quad \theta\mu_1 \geq 0; \quad \Gamma \geq 2(v_2+\mu_2); \quad \Gamma \geq 2(v_1^c+\mu_1).$$

Therefore, we obtain: $\frac{\partial(E[CS_1^f]-E[CS_1^c])}{\partial\theta} \leq 0.$ □

6.7. Description of Data in Computational Experiments

The price of an installation was based on the average installation price of Q1 and Q2 of 2011, $p_1 = 2.546€/W$ and $p_2 = 2.42€/W$ respectively.[5] We used a cost of installation roughly at 80% of the final price[6], $c_1 = 2.03€/W$. We vary the second period cost to display the inter-temporal difference in profit margins. In particular, we use values of c_2 ranging between 1.906 and $2.03€/W$.

This range of values explore the non-trivial regime we discuss in this paper. The lower bound is imposed by $p_1 - c_1 > p_2 - c_2$, otherwise the supplier would delay all its production to the second period when facing a flexible government. The upper bound is due to the condition that $c_1 > c_2$, otherwise most of the second period supply would be produced within the first period. Salvage value at the end of the horizon is set at $p_3 = 1.8€/W$, which does not affect the qualitative aspect of the simulation. Note that we must use a salvage value lower than the cost in period 2, c_2, otherwise the problem becomes trivial with a direct incentive to oversupply.

Given the total number of installations in 2009 equal to $3806MW$ and in 2010 at $7400MW$, we use the price and rebate level to estimate a simple linear sensitivity to rebate levels. Prices of solar panels at the time were $3.9€/W$ and $2.8€/W$ respectively for 2009 and 2010. The feed-in-tariff level was $0.43€/kWh$ in 2009 and between 0.33 and $0.39€/kWh$ in 2010. These tariffs reflect the sale price of electricity generated from the solar panel, which are fixed for 20 years from the installation of the solar panel. Considering the average annual output of solar panels in Germany ($876kWh/kW$) and the resulting 20-year stream of cash-flows discounted at 5% minus the upfront cost, we obtain a net present value of an installation at $0.79€/W$ and $1.13€/W$ in 2009 and 2010 respectively. Evaluating the rate of increased demand based on the increased economic benefit of a solar panel, we obtain $b = (7400-3806)/(1.13-0.79) = 10,571$. In other words, for every $€/W$ of

[5] International Energy Agency - Photovoltaic Power Systems Programme - Annual Report, 2012

[6] Seel et al. (2014)

subsidy we expect to obtain an additional $10,571MW$ of installations. In this section, we assume this sensitivity b to be the same over time. Using as a base the electricity price of $0.25€/kWh$ instead of the feed-in-tariff, we estimate the nominal demand for solar panels in the first and second half of 2011 at $\mu_1 = 1839MW$ and $\mu_2 = 3150MW$. Considering a target adoption of $\Gamma = 7500MW$, we use our model to find the optimal subsidy (feed-in-tariff) and the industry's supply level. For comparison purposes, the historical value of the feed-in-tariff in 2011 was $0.2874€/kWh$. We use $0.25€/kWh$ to be a baseline feed-in-tariff, which would lead to the nominal levels of demand μ_1 and μ_2. The optimal feed-in-tariff recommended by our model, depending on the demand uncertainty and costs, are in the range $[0.281, 0.287]€/kWh$ for the committed setting and $[0.281, 0.292]€/kWh$ for the flexible setting.

In the first set of simulations, Figures 2 and 3, we vary both the second period cost of production, $c_2 \in [1.906, 2.03]€/W$, and the magnitude of the demand uncertainty w_1 and w_2. We draw both w_1 and w_2 independently from a uniform distribution, ranging from $-A$ to A. Starting at a low value of $A = 10MW$ and increasing it to $A = 920MW$, we emulate various levels of standard deviation of demand uncertainty, σ. We restrict our simulation to values of A smaller than the average nominal demand μ, therefore preventing negative demands.