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**On The Portents of Peak Oil
(And Other Indicators of Resource Scarcity)**

by
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Abstract

Although economists have studied various indicators of resource scarcity (e.g., unit cost, resource rent, and market price), the phenomenon of “peaking” has largely been ignored due to its connection to non-economic theories of resource exhaustion (the Hubbert Curve). I take a somewhat different view, one that interprets peaking as a reflection of fundamental economic determinants of an intertemporal equilibrium. From that perspective, it is reasonable to ask whether the occurrence and timing of the peak reveals anything useful regarding the state of resource exhaustion. Accordingly, I examine peaking as an indicator of resource scarcity and compare its performance to the traditional economic indicators. I find the phenomenon of peaking to be an ambiguous indicator, at best. If someone announced that the peak would arrive earlier than expected, and you believed them, you would not know whether the news was good or bad. Unfortunately, the traditional economic indicators fare no better. Their movements are driven partially by long-term trends unrelated to changes in scarcity, and partially but *inconsistently* driven by actual changes in scarcity. Thus, the traditional indicators provide a signal that is garbled and unreliable.

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On The Portents of Peak Oil (And Other Indicators of Resource Scarcity)

“Portent: An indication of something important or calamitous about to occur.”

— thefreedictionary.com

1. Introduction

For better or worse, “peak oil” has gained considerable prominence as an indicator of growing resource scarcity. Since Hubbert (1956) introduced the hypothesis that oil production must reach a maximum and then fall into inexorable decline, economists have remained skeptical. That skepticism is largely due to the fact that Hubbert’s methods and predictions treat production of oil as an exogenous process divorced from market incentives. Nevertheless, the general public, and numerous scientists from disparate fields, remain clearly focused on the prospect of an impending and inevitable decline in oil production, and the notion that “peaking” manifests a scarcity that necessarily limits future economic growth. The peak, in other words, is undesirable because it ushers in a new age of painful, and potentially catastrophic, change.¹ Motivated by these sentiments, Campbell and Laherrere (1998) have declared that dating oil’s peak is more important even than dating its exhaustion. Consistent with this view, we have seen a virtual tournament of among analysts who have attempted to date the peak.²

¹ See, for example, Leigh (2008), Bardi (2005), and Campbell and Laherrère (1998).

² Recent attempts to date the peak include Wood, Long, and Morehouse (2003), Bartlett (2000), Duncan and Youngquist (1999), and Korpela (2006).

This paper considers the economic implications of peak oil. However, we take a somewhat broader view of the peaking phenomenon than what is represented by the Hubbert Curve. Any well functioning market economy, endowed with a limited amount of an exhaustible resource, will arrange production through time according to prevailing economic incentives that reflect market fundamentals such as the size of the resource stock, the cost of production, discount rates, the strength of current versus future demand, and the availability of substitutes. In contrast to the Hubbert Curve, where the peaking phenomenon is a physical imperative caused by “running out” of the resource, the forces that regulate production in a market economy might cause production to fall due to insufficient demand, rather than insufficient supply.

Obviously, it is the interaction of supply and demand that determines the equilibrium price path in a market economy, as well as consequent variations in the rate of production. Taking market forces into account does not avoid the inevitable peak, of course, but since both sides of the market (demand as well as supply) are integral to the rate of production, the peaking phenomenon generated within a market economy presumably conveys more information than what is implied by the Hubbert Curve. With this in mind, it seems at least possible that “peak oil” might serve as some kind of useful economic indicator of scarcity. Therefore, despite economists’ justifiable rejection of the Hubbert Curve, they may nevertheless reasonably ask whether the phenomenon of peaking within a market economy reveals something important about the state of resource scarcity—something that would not be revealed by market prices or resource rents, for example.

The objective of this paper is therefore to explore whether peaking provides a useful economic indicator of scarcity. The criterion I apply to make that determination is very basic and follows Brown and Field (1978), who set forth two criteria by which any indicator of resource scarcity might be judged. The first states quite simply that:

A minimum condition (for a good index of scarcity) is that the index go up when underlying determinants shift to increase actual or expected demand for the resource relative to actual or expected supply.

The second criterion goes quite a bit further:

It would be much more useful, furthermore, if it were possible to distinguish the contribution to the changes in the index made by each important determinant of demand and supply shifts.

The second criterion is notable because it requires the indicator to signal not just that a change has occurred, but also the source of the change. Although this may be a desirable characteristic of an ideal indicator, in practice it seems beyond reach, as I argue later. Thus, I will focus on Brown and Field's first criterion, which refers to an exercise in comparative statics and expresses the idea that an indicator of scarcity should provide reliable directional signals of unexpected changes to the balance of supply and demand.³

The rest of the paper proceeds as follows. In section 2, I contrast the two explanations of peaking behavior: first the approach built upon the Hubbert Curve, followed by a simple formulation of Hotelling's equilibrium production path. The usefulness of peaking as an indicator of scarcity is examined in Section 3. In Section 4 I assess the usefulness of the traditional economic indicators of scarcity, and discuss a

³ Some might prefer the term "comparative dynamics" as used by Oniki (1973) to describe analysis of the impact of parametric changes on the entire equilibrium path.

popular misconception that stems from the previous literature on the subject and offer a new interpretation of that literature. The findings of the study are summarized in the concluding Section 5.

2. Peak Oil: Hubbert vs. Hotelling

A peak rate of production figures prominently in all models of resource extraction. However, the reason for the peak, and its timing, vary according to the nature of the model. I illustrate this below, where Hotelling's peak is defined and clearly distinguished from Hubbert's peak.

Hubbert's Peak

The Hubbert Curve is built upon the assumption that production is distributed continuously through time according to a bell-shaped curve, which we denote by the function $q_t = URR \times f(t)$, where q_t represents the production rate at time t , URR represents the volume of ultimate recoverable reserves (assumed to be fixed and known), and where $f(\cdot)$ represents a normal probability distribution with mean T and standard deviation σ —which are parameters to be estimated.⁴ In addition to the presumed value of URR, the data which support estimation consist (minimally) of the production rate at t_1 (q_1) and cumulative production to t_1 (denoted Q_1). Hubbert's original sketch, which projects the eventual decline of global oil production, is shown in the upper panel of Figure 1. The lower panel is a generalized sketch that ties our terminology to Hubbert's construct.

⁴ This is a stylized formulation of the Hubbert Curve for illustrative purposes only. Most practitioners refer to $f(\cdot)$ as a time-series function rather than a probability distribution, but analytically they are the same. Many variations are possible and many have been employed. Hubbert (1967) actually applied the logistic distribution and many researchers have followed that lead. The assumption of symmetry is sometimes relaxed. The domain of the distribution can be specified in terms of number wells drilled, rather than time, as in Reynolds (2002), but unlike the Hubbert Curve, that approach produces something roughly akin to a production function, not a production forecast.

Because the maintained assumptions are quite strong, and although the given information (q_1 , Q_1 , and URR) is sparse, it is then possible to determine T and thus predict the onset of peak production. To demonstrate the procedure, we note first that the integral of q_t over the entire domain is identically equal to URR, and that peak production occurs at time T , the mode of $f(\cdot)$. We also note that Q_1/URR measures the proportion of the resource that has already been depleted, which according to the standardized normal distribution implies:

$$\Pr\left(z \leq \frac{t_1 - T}{\sigma}\right) = \frac{Q_1}{URR},$$

Or equivalently:

$$\frac{t_1 - T}{\sigma} = z_\alpha, \tag{1}$$

where $\alpha = Q_1/URR$, and z_α marks the lower tail of the standard normal distribution.

We also know that current production relative to ultimate recovery (this ratio is denoted β) gives the height of $f(\cdot)$ at t_1 . Thus:

$$\frac{q_1}{URR} = \beta = f(t_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{t_1 - T}{\sigma}\right)^2}. \tag{2}$$

After substituting for σ in (2) using $\sigma = (t_1 - T)/z_\alpha$ from (1), the implied value of T is obtained:

$$T = t_1 - \frac{z_\alpha}{\beta} \times \frac{1}{\sqrt{2\pi}} e^{-z_\alpha^2}. \tag{3}$$

The predicted date of peak oil (T) will be highly sensitive to the presumed value of ultimately recoverable reserves (through the impact of URR on both z_α and β). This

is problematic for economists since the volume (and timing) of ultimate recovery presumably depends upon price—which in turn depends upon demand, interest rates, and the cost of production—none of which are incorporated here.⁵ There is no assurance in Hubbert’s model that the projected rates of future production will actually clear the market. Although the prediction is simple, it is not credible due to neglect of these fundamental economic factors. Empirical tests of this procedure performed by Brandt (2007) and Nehring (2006), encompassing numerous global petroleum basins, failed badly in predicting the peak, which reinforces economists’ theoretical objections to the underlying method.⁶

It is possible to proceed along similar lines without presuming to know the volume of ultimately recoverable oil, at least if there are sequential observations available on production. For example, suppose that q_1 and q_2 represent the historical rates of production observed at two distinct dates, and let Q_1 and Q_2 represent the cumulative production volumes associated with those dates. The problem of calculating T is then reduced to identifying the unique normal distribution that conforms to these two points on the density function. As in the previous case, there are two equations and two unknowns, which gives a unique solution for T and σ . Once those values are

⁵ Proponents of the Hubbert Curve have an entirely different perspective on this point. Aleklett and Campbell (2003), for example, refer to depletion models that include things like investment, demand and supply, and other economic factors as the “flat-earth approach,” and claim that “the malign influence of doctrinaire economics” acts as an obstacle that stands in the way of efforts to date the peak.

⁶ The Hubbert Curve might have been forgotten altogether but for the fact that Hubbert’s 1956 prediction that U.S. oil production would peak around 1970 was famously borne out. It should also be noted (but usually is not) that the predicted volume of oil to be produced at the peak was 37% too low, and that Hubbert’s predictions regarding coal and natural gas ran badly amiss. Hubbert predicted that U.S. oil production would peak at 3 billion barrels per year; actual production in 1970 was 4.1 billion barrels. Hubbert predicted that U.S. gas production would peak at 14 trillion cubic feet per year in 1973; actual production was 21 trillion cubic feet in 2009. Hubbert predicted that global coal production will peak in 2150 at about 6.4 billion metric tonnes; actual production reached that level in 2007 and is still growing rapidly. (Actual production data are from the *BP Statistical Review of World Energy*, 2010 edition).

obtained, one solves for the volume of ultimately recoverable reserves by integrating the distribution. It is important to realize, however, that although URR is not assumed to be directly observable here, none of the structural assumptions of the model have been relaxed: ultimate recovery is still determined independently of the relevant economic variables.

In most applications of the Hubbert Curve, a time-series of *many* production data points is available, which means the two-parameter model is over-identified. Thus, some type of estimation criteria must be specified to identify the parameter values that minimize the fitted errors. Empirical attempts to apply the model in this fashion have revealed the poor quality of estimates (and predictions) that result. For example, Harris (1977) demonstrated that a simple OLS regression fit to Hubbert's data produces an estimate of URR that falls short even of the volume of already known discoveries—which implies that the volume of future discoveries must be negative. Cavallo (2004) reexamined Hubbert's data and found that the historical production curve does not reliably distinguish between alternative values of URR; the regression R-squared of Hubbert Curves with URR ranging between 150 and 600 billion barrels varies only in the third decimal point. Cleveland and Kaufmann (1991) also document the difficulty of estimating URR based on the goodness-of-fit of historical data to the Hubbert Curve.

Hotelling's Peak

Hotelling (1931) pioneered the *economic* study of peak oil, although his work (and most of the related models that have ensued) are usually not described in those

terms.⁷ Because the Hotelling model is well known to many economists, only a cursory review of the simplest possible application is provided here.⁸ More elaborate and realistic versions of the Hotelling model have been developed, and, as Gordon (2009) emphasizes, those extensions undoubtedly provide better empirical predictions of real-world production and price trends. However, our goal is not to predict the future, but to determine conceptually whether the peaking phenomenon provides a consistent indicator of scarcity. As we will show, peak oil fails this simple test even in models where the economy is deliberately oversimplified and the depletion phenomenon is entirely straightforward. Adding further complexity (realism) to the model would only increase the degree of ambiguity that surrounds the peaking phenomenon.

Let there be a fixed volume of the depletable resource (R) available at time t_0 . The unit cost of production (C) is constant through time, which implies no variations in resource grade and no technological change. The relevant discount rate (r) also remains constant through time. Demand for the resource is given by a constant elasticity demand function and the quantity demanded at any given price is assumed to grow exogenously at the rate g —which may represent the combined force of population growth, economic development, etc.⁹ Thus, the demand function can be written as:

⁷ Holland (2008) is one of the very few authors who have explored the timing of peak oil within the Hotelling framework. Greene, Hopson, and Li (2006) also investigate peaking behavior in the context of an economic transition from conventional to unconventional resources. Although Farzin (1995) does not specifically focus on the production peak, he does explore the relationship between a declining (or increasing) production trend and other measures of resource scarcity. Without embracing Hotelling's full equilibrium framework, various authors have shown via *ad hoc* specifications that economic variables improve the fit of the Hubbert Curve to historical data; see for example Bopp (1980), Kaufmann (1991), Cleveland and Kaufmann (1991), and Kaufmann and Cleveland (2001).

⁸ Readers who are interested in the many extensions to the basic model are referred to Hotelling (1931), Herfindahl (1967), Solow (1974), Levhari and Liviatan (1977), Devarajan and Fisher (1981), Krautkraemer (1998), Farzin (1992), etc.

⁹ I have also investigated models with linear demand and can say that the results reported here are robust to the form of the demand function.

$$Q_t^d(P_t) = K \times P_t^\varepsilon \times e^{gt}$$

Moreover, a perfect substitute for the depletable resource (i.e., a backstop technology) is available in unlimited quantities at constant unit cost (B). The cost of the backstop is assumed to be known but may be relatively high.

Under these conditions, and assuming that markets are competitive and that a full set of futures markets exists, a unique inter-temporal equilibrium exists and is characterized by the price path that satisfies the following three conditions:

- (i) $P_t - C = (P_0 - C)e^{rt} \quad \forall t \in (t_0, T)$ [no inter-temporal arbitrage]
- (ii) $P_T = B$ [transition to backstop at time T]
- (iii) $\int_{t_0}^T Q_t^d(P_t) dt = R$ [resource exhaustion at time T]

Because production of the resource is positive only during the finite interval $[t_0, T]$, it follows that a peak rate of production must exist. Although a closed-form solution does not exist for the date of the peak, it can easily be computed once specific functional forms and parameter values are specified. The timing of the peak will generally depend upon the fundamental economic factors that describe the economy. The peak may come at mid-course (as predicted by Hubbert), or early, or late in the course of exploitation—all depending upon the elasticity of demand, the economic growth rate, the discount rate, the initial volume of resource available, and the cost of producing the depletable resource relative to that of the backstop technology.¹⁰ Indeed,

¹⁰ Gordon (1967) and Levhari and Liviatan (1977) were among the first to characterize the influence of various economic factors on the shape of the equilibrium path.

in more elaborate models that incorporate exploration, technological change, etc., the rate of production may peak more than once along the equilibrium path.

3. Peaking as an Indicator of Scarcity

I illustrate the relationship of the Hotelling peak to underlying economic fundamentals using a series of examples. In each case, the long-term elasticity of demand is assumed to be -0.35. The discount rate is 10%, and the initial volume of the resource is assumed to be 4,000 units. The unit cost of production is \$20 for the resource versus \$100 for the backstop technology. In what I will refer to as the *Benchmark* scenario, the growth rate of demand is taken as 1.5% per annum, and the demand function (intercept) is calibrated such that, if the resource were priced at the cost of production (i.e., \$20), demand would exhaust all available supply in roughly 23 years. The case is hypothetical, of course, and the unit of physical measurement is arbitrary, but the general outline perhaps paints a picture of the world oil market that is not implausible. All costs, growth rates, and reserve-to-production ratios are certainly within the realm of historical experience.

This particular set of assumptions leads to the equilibrium price and production paths shown in Figure 2.¹¹ The price starts at \$26.61 (30% above cost, which represents the scarcity rent or “user cost” of the resource at that time). Over the course of 26 years and 3 months, the price (red line) rises to \$100, which is the cost of the backstop, and given the strength of demand at these prices, the total resource endowment is produced and exhausted just as the backstop technology becomes economic. Beyond that point, the price no longer increases due to the unlimited supply

¹¹ The trajectories of price and production shown in the figure were computed in Excel using a discrete-time approximation to equations (i) – (iii), where each time-step represents one month.

of the backstop technology. Production, on the other hand, first rises then falls. Peak production occurs after 36% of the original resource has been depleted—roughly one-third of the way into the period of exploitation. If we were to rely on the peak to signal impending exhaustion, it seems that substantial warning is supplied; it certainly provides more than a last-minute indication. In that respect, this particular economic scenario of depletion is not much different than the standard Hubbert result, where the peak comes around the half-way mark.

It should not be surprising to find that different results can be obtained by varying the underlying economic parameters.¹² Indeed, it is perhaps heartening (at least on first examination) to find that very slight changes in the underlying economic factors tend to cause rather dramatic changes in peaking behavior—for if the peaking phenomenon did not register changes in the underlying economic factors, it could hardly serve as an indicator of the degree of scarcity. This is illustrated by the next two scenarios (*High-Growth* and *Low-Growth*) which deviate from the *Benchmark* case only in terms of the assumed economic growth rate. In the *High-Growth* case (Figure 3), the rate of growth is increased from 1.5% to 2.75%—a small change well within the range of historical experience. The resulting impact on price is also relatively small, it now starts at \$28.50 instead of \$26.61, but the impact on peaking behavior is dramatic: the peak is now delayed to the very end of the exploitation period when none of the resource remains. The *High Growth* outcome contrasts even more sharply with the *Low Growth* case (Figure 4). When the growth rate is reduced to 0.5% (also within the realm of historical experience), the peak occurs at the very outset of exploitation, when 100% of the

¹² Gordon (1981) showed, for example, that the shape of the equilibrium path will depend on whether demand is stationary, rising, or falling over time.

resource remains.¹³ From that point on, production is all downhill. The results of these various scenarios are summarized in Table 1.

Does the earlier peak of the *Low-Growth* case signify greater scarcity? Certainly not in a purely physical sense because the same resource endowment is common to all three scenarios. Indeed, since demand is always smaller (holding prices constant) in the *Low-Growth* case, it would be reasonable to claim the resource is in fact less scarce. The apparent implication is that peaking provides an inverse indicator of resource scarcity: the more economically abundant is the resource, the earlier will the peak arrive. Unfortunately, however, the behavior of the peak is not so simple or consistent, as further analysis reveals.

Consider, as an alternative to the *Low Growth* scenario, the impact of a uniform and permanent 25% reduction in demand, as might occur if consumers were suddenly to adopt a more conservation-oriented lifestyle.¹⁴ Just as in the *Low-Growth* case, this *Lifestyle* scenario takes pressure off the finite resource base relative to the Benchmark case and reduces the degree of scarcity. The impact on peaking behavior, however, is in the opposite direction (Figure 5). Here the reduction in scarcity causes the peak to be delayed, arriving not until 48% of the resource has been depleted, as opposed to 36% in the Benchmark case (and 0% in the Low Growth case). Thus, a demand reduction may either advance or retard the peak—depending on the particular circumstances of the reduction. Apparently, the timing of the peak is an inconsistent indicator of changes in the underlying degree of resource scarcity.

¹³ In Hotelling's original specification, with stationary demand and constant cost, the peak always occurs before any depletion has occurred. Levhari and Liviatan (1977) and Gordon (1981) demonstrate how relaxing those assumptions gives rise to varied production trends.

¹⁴ This is effected, holding all else constant, by a 25% reduction in K , the intercept of the demand function.

The next case to consider is the impact of diminished expectations regarding the remaining physical endowment of the resource. If, for example, the initial volume of resource were reduced by 25% (falling from 4,000 to 3,000) units, that certainly represents an increase in scarcity. Under this *Resource Disappointment* scenario (Figure 6), in which all other parameters are held constant at *Benchmark* values, the peak arrives substantially earlier, after only 21% of the (diminished) volume of resource has been consumed. But, this comparative static reaction conflicts with the *Low Growth* case considered previously: “slower growth = less scarcity = *earlier* peak” vs. “less resource = more scarcity = *earlier* peak.” The difference cannot be ascribed to market myopia or mistaken expectations since all of these scenarios reflect rational equilibrium behavior predicated upon perfect information and a well functioning set of futures markets.

Before leaving the *Resource Disappointment* case, it may be worth noting that the immediate price impact of what is a fairly spectacular downward revision to the resource endowment is muted: although the resource stock was reduced by 25%, the price of the resource rises initially by only 15% (relative to the *Benchmark* case) and production declines by only 5%. Part of the reason for the muted response, of course, is that it is more economical (due to the impact of discounting) to postpone much of the adjustment in consumption to the future. This does not represent myopia, but the efficient functioning of a market economy that weighs future benefits against present costs. The potential relevance to the oil market in the real world is the idea that a sudden perception, say, that some portion of Saudi oil reserves are “illusory” is not so

likely to trigger a sensational increase in the market price of oil, contrary to what some observers have suggested as a cause of the 2008 oil price spike.¹⁵

Further Ambiguity

Based on the results thus far, we have an impression that the timing of the peak does not reliably distinguish between greater or lesser scarcity. This conclusion is reinforced if we consider how cost changes affect peaking behavior. Imagine, for example, that due to a one-time technological breakthrough the cost of producing the resource could be reduced to \$15 (from \$20 in the Benchmark case), which by most definitions would reduce the economic scarcity of the resource. The impact of this breakthrough on the market equilibrium (Figure 7) would be to reduce the initial price to \$22.42 (from \$26.61). Notice that the entire cost reduction is not passed through since the market price incorporates the resource rent as well as the cost of production—and resource rent rises due to the increased cost advantage relative to the backstop technology. However, the peak production now arrives after only 21% of the resource has been consumed—much earlier than in the *Benchmark Case* where the peak arrived after 36% had been consumed. Thus, this result seems to conform to the “decreased scarcity=earlier peak” pattern that we encountered before. However, if technological progress is continuous, rather than one-shot, the pattern is again reversed. Let there be, in place of the one-shot cost reduction, an exogenous 3% annual reduction in the cost of producing the resource, with all else as in the *Benchmark case* (Figure 8). Like the previous cost-saving example, by most accounts this would signify decreased economic scarcity. However, the impact of this type of technological progress is not to

¹⁵ Here I am referring to the adjustment of price to its new long-run equilibrium path. The short-run reaction, although not persistent, would be more pronounced due to the lower short-run elasticities of both demand and supply.

advance, but to delay the peak, which now occurs only after 40% of the resource has been produced (as opposed to 36% in the *Benchmark* scenario and 21% in the *Technological Breakthrough* scenario). It may be that increased scarcity is indicated by a later peak, but sometimes it is just the reverse.

4. Traditional Indicators of Scarcity

I have demonstrated that peak oil fails Brown and Field's minimum condition for a useful indicator: it does not always "go up" or "down" in a consistent manner. Do the traditional economic indicators of resource scarcity (i.e., unit cost, resource rent, and market price) fare better? Despite the rather extensive literature that has pursued this question, the answer is not altogether clear. Whereas previous studies have been widely interpreted as demonstrating the failure of the traditional indicators, I argue that the basis for that conclusion is open to question and requires clarification and reinterpretation.

A Reinterpretation of the Literature

To be sure, there have been many investigations of the dynamic behavior of the traditional indicators. But that literature follows a different tack: one where the comparative static impact of changes in scarcity have not been the subject of inquiry. What has been investigated is a very different issue: how the traditional indicators progress as the economy moves along a given equilibrium depletion path.¹⁶ A rising equilibrium price trajectory can easily be rationalized, but so can a U-shaped curve, as in Slade (1982). Moreover, given the range of plausible variation in the structural

¹⁶ Analyses of this type are numerous and include Fisher (1979), Slade (1982), Devarajan and Fisher (1982), Halvorsen and Smith (1984), Livernois and Uhler (1987), Norgaard (1990), Reynolds (1999), Cleveland and Stern (1999), and Farzin (1995), to name but a few.

factors that enter into these models (e.g., technical change and exploration), almost any pattern or trend involving price, cost, and resource rent might be encountered as a given economy progresses along its equilibrium depletion path.¹⁷

These studies give a correct picture of how a given equilibrium unfolds through time, but they have been misinterpreted, in my opinion, as also addressing the impact of comparative static changes in scarcity. Thus, Fisher (1979) finds that “cost and rent are sometimes well behaved indicators of scarcity, sometimes not. ... Where cost moves in the right direction [*as the economy progresses along the equilibrium depletion path*], rent does not, and *vice versa*.”¹⁸ Likewise, Farzin (1995) notes that the search for an appropriate indicator of scarcity is complicated by the fact that, in numerous cases, the three traditional indicators will not move in concert along the equilibrium path. Halvorsen and Smith (1984) observe that “increasing physical scarcity may be associated with either increases or decreases over time [*along the equilibrium path*] in any or all of the proposed measures.” And Cleveland and Stern (1999) opine that along a U-shaped equilibrium price path, the market price might provide a false signal of reduced scarcity.

I argue that the reported failure of these indicators is somewhat peremptory because the question of what is to be signaled has not yet been properly framed. What is the policy relevance or prescriptive importance of changes in scarcity indicators that are observed as the economy progresses along a given equilibrium path? We cannot say these changes indicate *unexpected* shifts in the underlying fundamentals, or that they mark a departure from the existing equilibrium. They are not the result of a

¹⁷ For example, see Halvorsen and Smith (1984), Farzin (1992), and Krautkraemer (1998).

¹⁸ I have inserted the italicized phrase to clarify the context.

comparative static shock to the economy. Rather, the evolving measures of scarcity are the predetermined result of society's choice of an optimal depletion path (assuming that markets function well and without externalities, which is typical of this literature). Within that framework, the degree of resource scarcity observed at each point along the equilibrium path is exactly the level that is required to maximize social welfare. So what if an indicator is interpreted to mean that scarcity has increased from yesterday to today? Society chose that path of exploitation (and increasing scarcity) and any deviation would only impose unnecessary costs on the economy. In other words, it would not make sense to interpret the trend of increasing resource scarcity that is experienced along a given equilibrium path as an indication that we have become poorer, or as an invitation for the government or some other intervener to take actions designed to slow the rate of depletion or otherwise alter the course of events.

In this respect, the discussion of scarcity indicators may benefit from a closer connection to the literature on economic sustainability. A sustainable path, as described by Solow (1992), is one that allows every future generation the option of being as well off as its predecessors. Our duty to future generations is not to bequeath any particular thing (like a specific mineral deposit), but to preserve a generalized capacity to produce economic well-being, which requires replacing used-up resources with other forms of produced capital.¹⁹ As we move along any equilibrium depletion path, making such investments as we go, this generalized level of welfare does not change from year to year, or generation to generation. The level of welfare *would* be affected, however, if there were an unexpected change in resource stocks, technology,

¹⁹ To sustain the present level of consumption requires net investment in produced capital equal to the sum of resource rents from that portion of the finite resource that is extracted each period. See Hartwick (1977).

or demand; that is to say, if there were a comparative static change in the economic fundamentals.

I understand this to mean that not all changes in scarcity indicators are created equal. Changes caused by unexpected shocks (comparative static effects) must be interpreted differently than changes that simply record the economy's continued progress along a given equilibrium path, as indicated by Figure 9. The upper panel demonstrates the comparative static shift to a new equilibrium path (with higher prices and lower welfare) due to the unexpected loss of resource endowment. The lower panel demonstrates the movement to higher prices along one equilibrium path (with no effect on welfare) that accompanies scheduled production. This poses the practical problem of how the two types of change can be separated or distinguished, and that is where the equilibrium models have an important role to play. If it is established, for example, that resource rent will grow in equilibrium at the rate of interest, then comparative static changes in the underlying fundamentals would be indicated by any deviation from that trend. It is not the change in the scarcity indicator that signals a favorable or unfavorable development, it is a deviation from the expected change that matters.

Apparently, such deviations are frequent based on the lack of success reported in fitting historical time-series data to the Hotelling model.²⁰ These failures should not necessarily be interpreted as rejections of the Hotelling framework, although they are often characterized that way. As Swierzbinski and Mendelsohn (1989) and Krautkraemer (1998) point out, the flow of information constantly generates

²⁰ For example, see Barnett and Morse (1963), Smith (1979), Slade (1982), Heal and Barrow (1980), Smith (1981), Farrow (1985), and Halvorsen and Smith (1986).

unanticipated changes in expectations regarding relative scarcity of the resource, which means that the historical time series data should not be expected to conform to an equilibrium path, even though expected future prices may do so.

What follows from this discussion is that the usefulness of a given indicator of resource scarcity is determined by its ability to accurately signal comparative static shifts, not by its evolution along an equilibrium path. In other words, I am not bothered that Farzin (1992) has found conditions under which resource rent may *fall* as the economy progresses along a given equilibrium depletion path, as long as that same indicator would rise if some portion of the remaining stock were unexpectedly removed. This distinction takes us back, of course, to Brown and Field's first criteria, which is focused on an indicator's ability to consistently signal comparative static changes in the underlying fundamentals.

Unfortunately, it is not so easy in practice to recognize the shocks to long run equilibrium trajectories as they occur, or to separate the comparative static movements from the evolutionary trend. What the previous literature has established is that the traditional indicators of scarcity are partially driven by long-term trends that are unrelated to scarcity. Thus, movements that truly signal unexpected changes in scarcity (i.e., movements away from the trend) may be difficult to isolate and identify.

Comparative Static Behavior of the Traditional Indicators

Even if it were possible, with sufficient research, to isolate comparative static movements in the traditional indicators from their evolutionary trend, those movements would not, unfortunately, provide consistent or reliable signals of underlying changes in scarcity. This point is new to the literature and requires further elaboration.

The comparative static properties of the traditional indicators are simpler in some respects than the previous literature might suggest. Unit cost, for example, is a direct measure of the inputs required to produce the resource, and any increase in unit cost implies that, holding output in the rest of the economy constant, there is a reduced capacity to produce the resource in question. Or, if the level of resource production is to be maintained, then production of something else somewhere else must be sacrificed. Thus, unexpected changes in unit cost provide unambiguous signals of changes in the underlying degree of scarcity. However, as Krautkraemer (1998) and others have observed, unit cost is driven by technology, not forward looking, and therefore cannot be counted on to signal changes in relative scarcity that are triggered by shifts in demand.

Resource rent, the second traditional indicator of scarcity (and one that is preferred by a number of resource economists), unfortunately provides comparative static signals that are not well behaved.²¹ On the one hand, resource rent represents the shadow price of the resource constraint and (holding all else constant) varies directly with the degree of physical scarcity.²² Thus, unexpected variations in the resource stock will be reflected by contemporaneous changes in resource rent: greater scarcity creates higher rent. On the other hand, other factors (like technical change and the strength of demand) also impact the shadow price of the resource and in certain cases will exert a countervailing influence that garbles the signal and creates ambiguity.

For example, recall the *Continuous Innovation* scenario from Section 3, where the *Benchmark* equilibrium was perturbed by introducing an exogenous 3% annual

²¹ See for, example, the discussion in Devarajan and Fisher (1982).

²² Fisher (1979) expresses a presumption that this result would hold even if there were not a complete set of futures markets for the resource in question.

reduction in unit cost (less scarcity). The immediate impact is to increase resource rent since cheaper extraction makes each unit of the resource more valuable relative to the backstop technology (see Table 1). We have an ambiguity because, in this case, it is a *reduction* in scarcity that triggers higher rents. (Recall that an increase in physical scarcity had a similar impact—see previous paragraph). Thus, diagnosing underlying changes in scarcity on the basis of comparative static changes in resource rent obviously requires additional, more specific information about the fundamentals.

The third indicator of scarcity, market price, is just the sum of the other two indicators. It therefore inherits whatever ambiguity they possess. If a factor that *increases* scarcity (*Resource Disappointment*) will trigger an increase in resource rent, then so will it increase the price. If another factor that *decreases* scarcity (*Continuous Innovation*) will also trigger an increase in resource rent, then so will it also increase the price. So, what are the implications of a comparative static increase in the price of the resource? Without more specific fundamental information, it is impossible to say.

Brown and Field's Second Criterion

Brown and Field's desire for an indicator that would signal not just a change in scarcity, but also the source of that change, is demanding and on close inspection probably unattainable. The reason for this is not specific to exhaustible resources; it applies also in the realm of renewable resources. Consider, for example, the market for a regular good, like wheat. Instead of an intertemporal equilibrium price path, the market is presumed to clear independently in every period (for simplicity, I ignore inventories). Figure 10 shows a succession of two such equilibria where both the price and quantity of wheat have increased during the given time interval. Taken by itself,

that information does not allow one to infer whether supply has *increased* or *decreased* from one period to the next. Only if the slope (or elasticity) of the supply curve is known can we distinguish the two possibilities. What we can say, based on the principles of market equilibrium, is that if price and quantity of wheat have both moved in the same direction, then the demand for wheat must have shifted (and in the same direction). We can also say that if price and quantity have moved in opposite directions, then supply of wheat must have shifted (in the same direction as quantity). But in neither case can we say how large was the shift, nor can we tell whether the “other” curve has increased or decreased during the interval.

As demonstrated in Smith (2009), knowledge of the elasticities of both supply and demand is required to say more. In Figure 11, which shows a more complete picture of the market for wheat, equilibrium moves from (p_1, q_1) to (p_2, q_2) when demand and supply are perturbed. The shift in demand, which is related to the combined effects of income, population growth, and other factors, is measured by the increase (holding price constant) from q_1 to \underline{q}_2 , or in percentage terms by $\Delta_D = \underline{q}_2 / q_1$. Although \underline{q}_2 is not observable, its value can be deduced based on an estimate of the elasticity of demand, as follows. The difference between \underline{q}_2 and q_2 represents a movement along the demand curve: $\underline{q}_2 / q_2 \approx (p_1 / p_2)^{\epsilon_D}$. This approximation is good for small price changes, and exact for all price changes if the elasticity of demand is constant. Making this substitution allows us to identify the demand shift: $\Delta_D = (q_2 / q_1) \times (p_1 / p_2)^{\epsilon_D}$. By similar means, the underlying shift in supply can be inferred from the presumed elasticity of supply: $\Delta_S = (q_2 / q_1) \times (p_1 / p_2)^{\epsilon_S}$. Thus, the observed prices and quantities must be

supplemented by knowledge of the respective elasticities if we are to discern the impact of a shock to a particular factor.

Even with a comprehensive set of historical market data (price and quantity) for the good in question it would be impossible to statistically estimate the elasticities in question without knowledge of movements in the concomitant variables (incomes, costs, etc.) that serve to identify each curve. Analysts study market fundamentals for at least two reasons. First, of course, is the fact that they wish to anticipate changes in prices before those changes are registered in the market. Second is the fact that, even after such changes are recorded, it is impossible to disentangle the causes solely on the basis of the observed movements in price and quantity. The punch line is that no simple indicator of increasing scarcity, or its causes, is available and examination of all the available indicators of industry conditions is needed.

5. Conclusion

Although the concept of peak oil originated outside the field of economics, the peaking phenomenon can be viewed as an integral part of an intertemporal equilibrium that reflects the influence of supply and demand. Placed in that context, where timing of the peak is endogenously determined by economic forces rather than nature, peaking is an event that potentially reveals meaningful information regarding the underlying market fundamentals. As part of the ongoing search for useful economic indicators of resource scarcity, it seems reasonable therefore to investigate the properties of the peak and compare its usefulness to that of the more familiar alternatives.

Unfortunately, as this paper demonstrates, peaking is an ambiguous indicator that provides inconsistent signals regarding resource scarcity. Even if it were

announced that the peak will arrive earlier than expected, and you believed it, you would not know whether the news was good or bad. However, we have also shown that the traditional indicators of resource scarcity (unit cost, resource rent, and price) fare no better. They provide inconsistent signals of real changes in the underlying degree of resource scarcity, and they also incorporate the confounding influence of long-term trends unrelated to changes in scarcity.

Thus, we cannot rely on any limited set of market indicators to gauge the status of our exhaustible resources. This conclusion echoes the view of Cleveland and Stern (1999), who suggest that in order to develop more effective forecasts of future resource scarcity we need to look beyond the indicators to the production technologies, natural resource bases, and market structures that determine the indicators. In other words, although the market fundamentals determine the indicators, the indicators do not reveal the fundamentals. That knowledge comes only from detailed study of the primary economic factors that lie further below the surface.

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Figure 1a: Hubbert's Worksheet (1956)

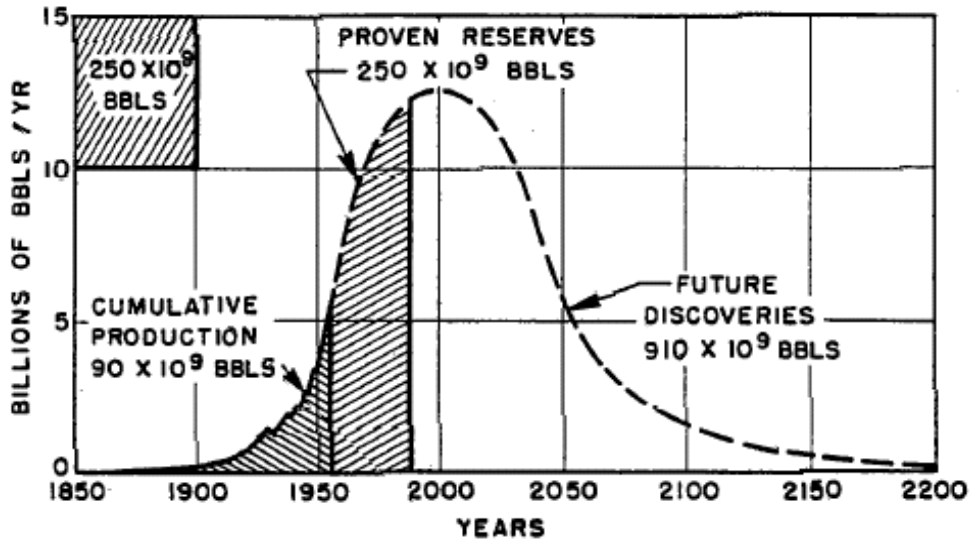


Figure 1b: Hubbert's Curve—General Scheme

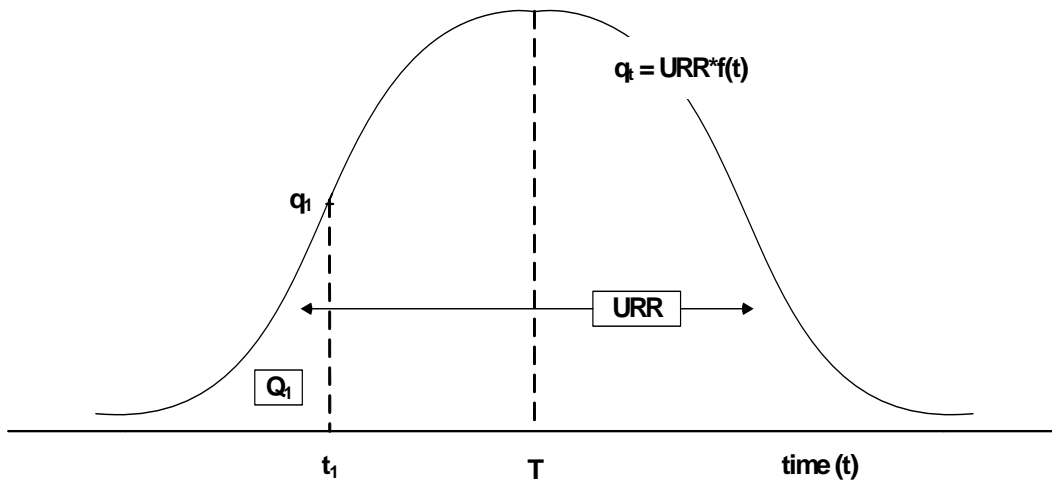


Figure 2: Benchmark Scenario (elasticity = -0.35, growth = 1.5%)

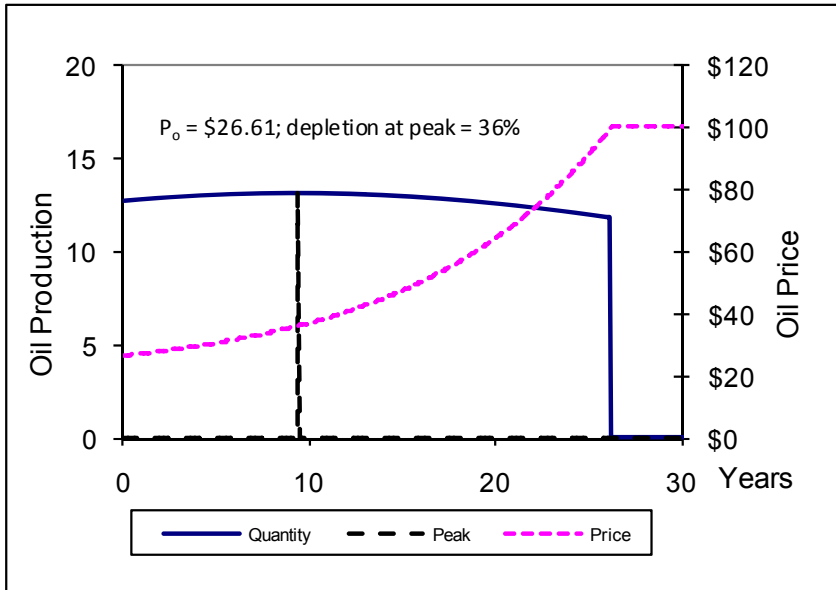


Figure 3: High Growth Scenario (elasticity = -0.35, growth = 2.75%)

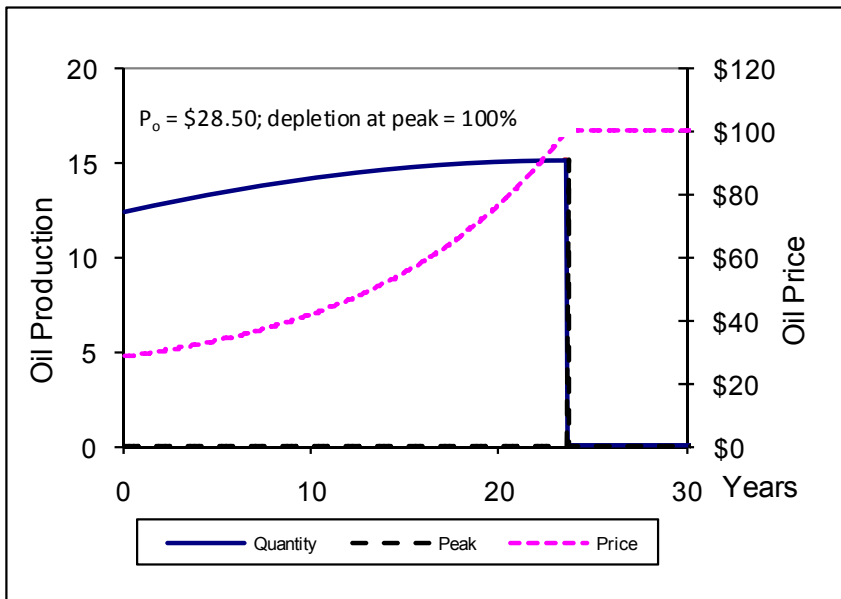


Figure 4: Low Growth Scenario (elasticity = -0.35, growth = 0.5%)

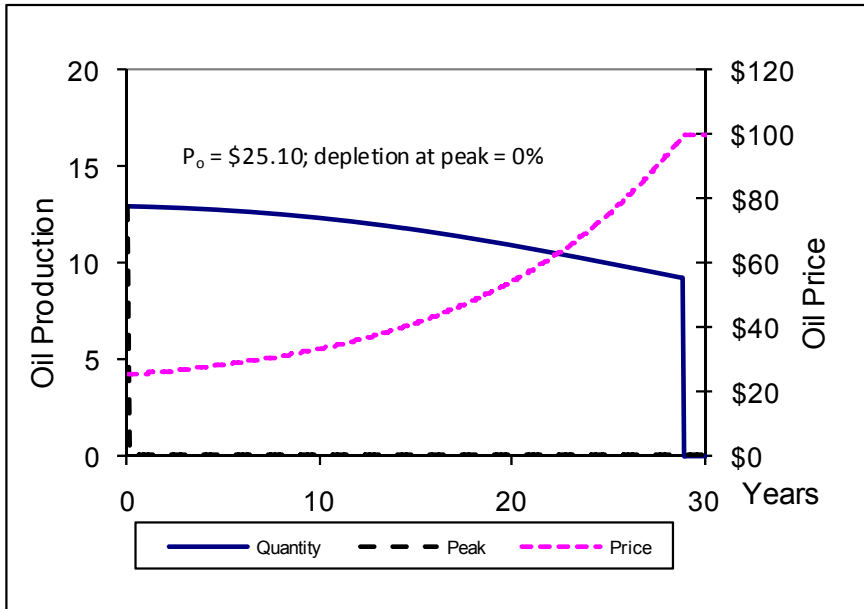


Figure 5: Lifestyle Scenario (25% demand reduction)

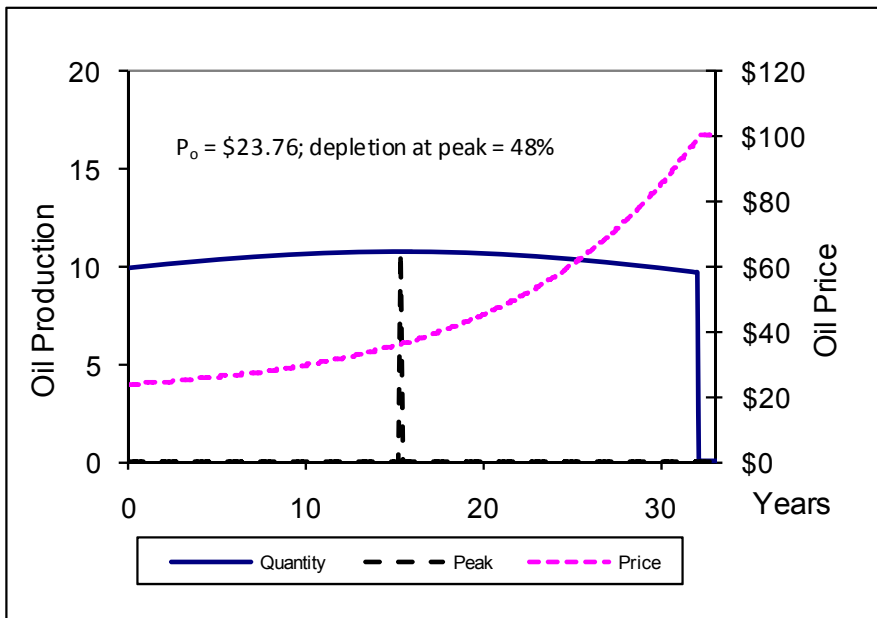


Figure 6: Resource Disappointment Scenario (elasticity = -0.35, growth = 1.5%)

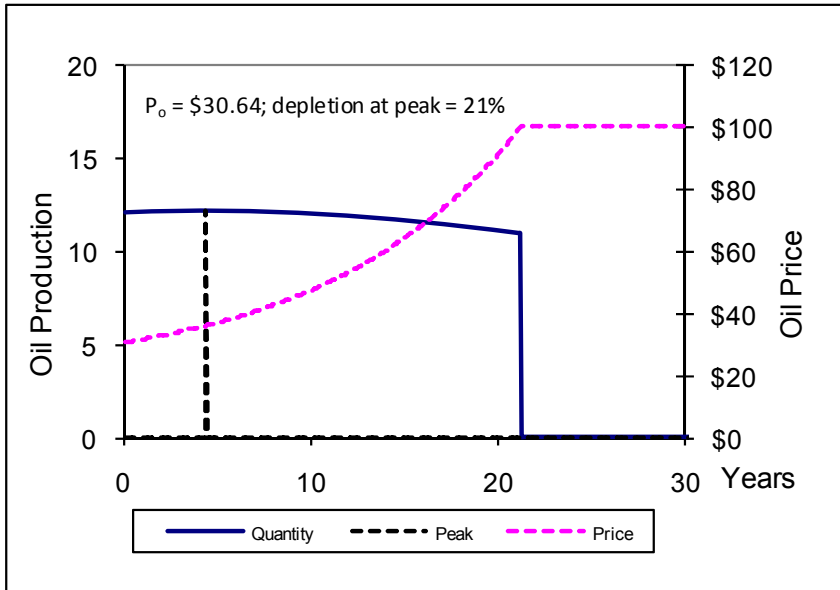


Figure 7: Technological Breakthrough Scenario (elasticity = -0.35, growth = 1.5%)

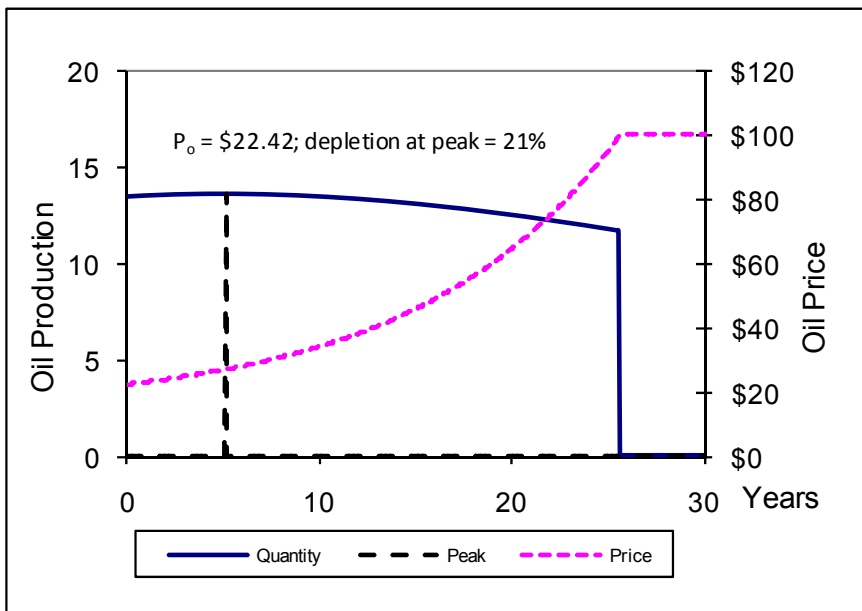


Figure 8: Technological Progress Scenario (elasticity = -0.35, growth = 1.5%)

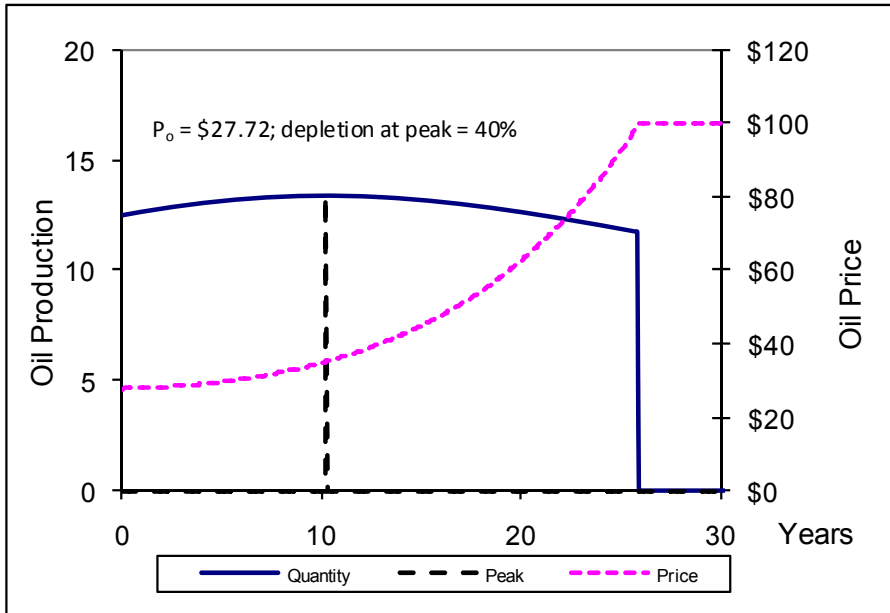
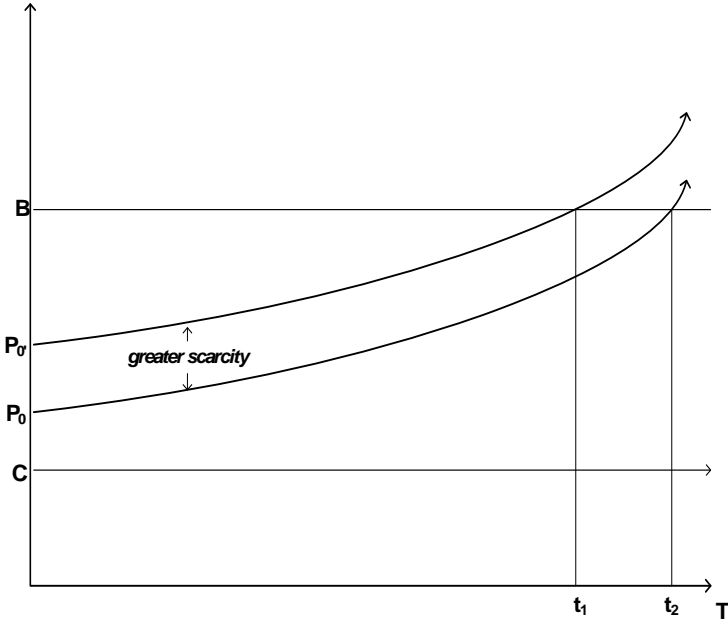


Figure 9: Comparative Statics vs. Equilibrium Dynamics of Scarcity

Panel A: Impact of Unexpected Reserve Loss



Panel B: Impact of Equilibrium Depletion

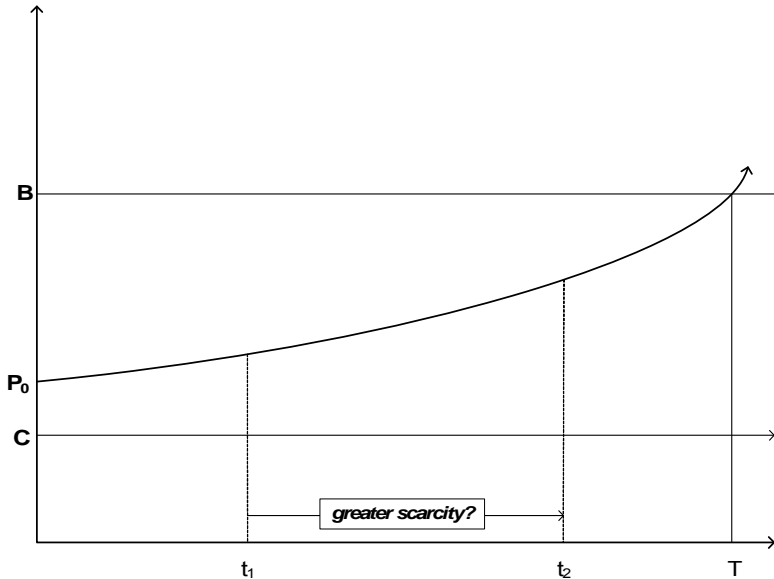


Figure 10: What Do Market Indicators Reveal?

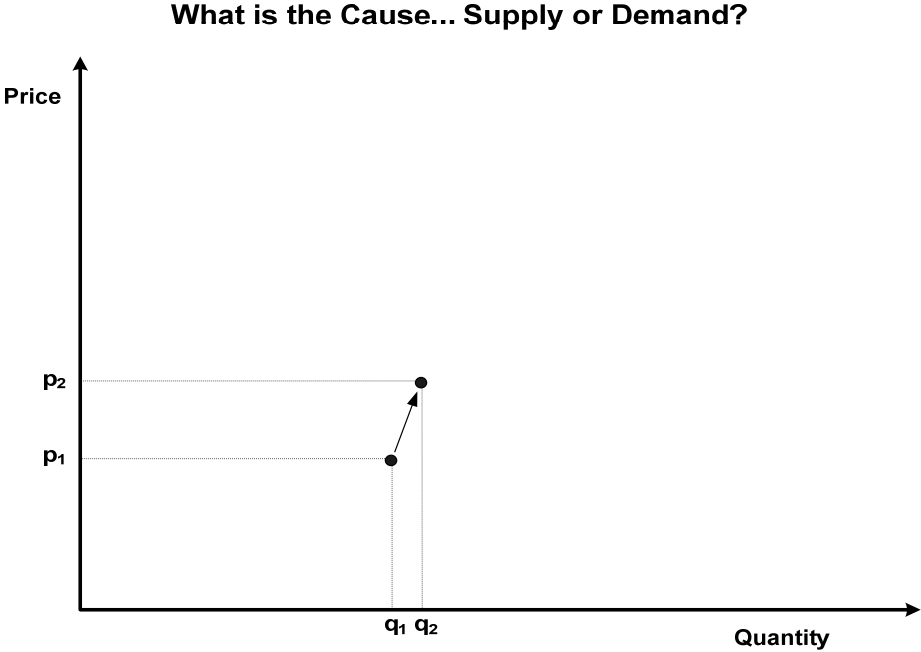


Figure 11: Decomposing the Change in Fundamentals

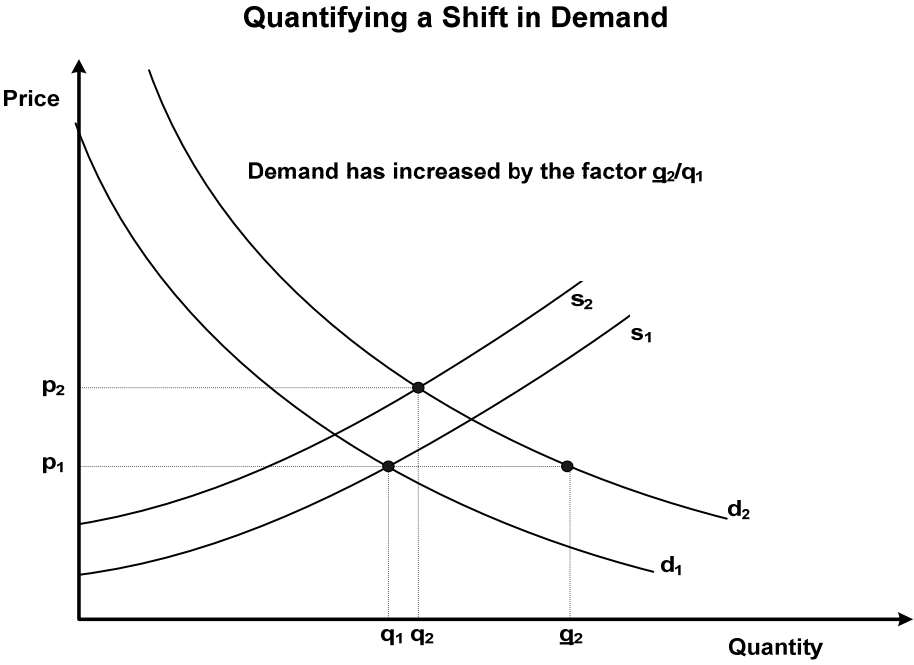


Table 1: Summary of Equilibrium Depletion Paths, by Economic Scenario

Economic Scenario	P_0	P_T	T	t_{peak}	t_{peak}/T	Q_{peak}/URR	r_{monthly}	r_{annual}
Benchmark	\$26.61	\$100.00	315	113	36%	36%	1.20%	15.39%
High Growth	\$28.50	\$100.00	284	284	100%	100%	1.24%	15.98%
Low Growth	\$25.10	\$100.00	348	0	0%	0%	1.15%	14.73%
Resource Disappointment	\$30.64	\$100.00	255	53	21%	21%	1.29%	16.60%
Cost Breakthrough	\$22.42	\$100.00	308	62	20%	21%	1.46%	18.98%
Continuous Innovation	\$27.72	\$100.00	311	123	40%	40%	1.17%	14.93%
Life Style Change	\$23.76	\$100.00	386	184	48%	48%	1.10%	13.98%

<p>Legend:</p> <p>P_0 = Price at initial time.</p> <p>P_T = Price at time when resource is fully depleted.</p> <p>T = Elapsed time before depletion occurs (in months).</p> <p>t_{peak} = Elapsed time before peak production occurs (in months).</p> <p>t_{peak}/T = Relative timing of peak production.</p> <p>Q_{peak}/URR = Cumulative depletion before peak production occurs.</p> <p>r_{monthly} = Average monthly price change.</p> <p>r_{annual} = Average annual price change.</p>
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