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by
Robert S. Pindyck

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by

Robert S. Pindyck
Massachusetts Institute of Technology

Neng Wang
Columbia University

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Abstract: What is the likelihood that the U.S. will experience a devastating catastrophic event over the next few decades – something that would substantially reduce the capital stock, GDP and wealth? What does the possibility of such an event imply for the behavior of economic variables such as investment, interest rates, and equity prices? And how much should society be willing to pay to reduce the probability or likely impact of such an event? We address these questions using a general equilibrium model that describes production, capital accumulation, and household preferences, and includes as an integral part the possible arrival of catastrophic shocks. Calibrating the model to average values of economic and financial variables yields estimates of the implied expected mean arrival rate and impact distribution of catastrophic shocks. We also use the model to calculate the tax on consumption society would accept to reduce the probability or impact of a shock.

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1 Introduction.

What is the likelihood that the U.S. will experience a devastating catastrophic event over the next few decades? Even if the probability is small, what does the possibility of such an event imply for the behavior of economic variables such as capital investment, interest rates, and equity prices? And how much should society be willing to pay to reduce the probability or the likely impact of such an event?

By “catastrophic event,” we mean something national or global in scale that would substantially reduce the capital stock and/or the productive efficiency of capital, thereby substantially reducing GDP, consumption, and wealth. Examples that we have in mind (you can come up with your own) include a nuclear or biological terrorist attack (far worse than even 9/11), a highly contagious “mega-virus” that spreads uncontrollably, a global environmental disaster, or a financial and economic crisis on the order of the Great Depression. Unlike more locally contained events such as Hurricane Katrina or the recent Asian tsunami, as terrible as they were, the events of concern to us would destroy part of the country’s (or the world’s) productive capital and raise future costs of operating the remaining capital.¹

Our approach to analyzing the economics of catastrophes differs considerably from the existing literature. We do not try to estimate the mean arrival rate and impact distribution of catastrophic events from historical data, nor do we use the estimates of others. Instead, we develop an equilibrium model of the economy and estimate these characteristics as a calibration *output* of our analysis. In effect, we are assuming that the calibrated characteristics of catastrophes are those perceived by firms and households, in that they are consistent with behavior, and thus with the data for key economic variables.²

Behavioral reactions to possible catastrophic events depend in part on preferences. Like

¹Those readers who are incurable optimists and/or have limited imaginations should read Posner (2004), who provides additional examples and argues that society fails to take these risks sufficiently seriously, and also Sunstein (2007). For a sobering discussion of the likelihood and possible impact of nuclear terrorism, see Allison (2004). In an excellent review article of Posner’s book, Parson (2007) points out the need for a general cost-benefit framework to address these risks in a consistent way.

²In related work, Russett and Slemrod (1993) used survey data to show how beliefs about the likelihood of nuclear war affected savings behavior, and argue that such beliefs can help explain the low propensity to save in the U.S. relative to other countries. Also, see Slemrod (1990) and Russett and Lackey (1987).

some other recent studies, we assume that households have recursive preferences, which involve three behavioral parameters: the rate of time preference, the index of relative risk aversion, and the elasticity of intertemporal substitution. There is little agreement among economists regarding the “correct” values for these parameters, but our calibration exercise provides insight into their plausible ranges and relative magnitudes.

We specify an *AK* model of production more general than those used by others in that it includes adjustment costs, so consumption and investment goods are not freely interchangeable and Tobin’s q can exceed one. Adjustment costs are crucial, and enable us to generate endogenous consumption-investment and consumption-wealth ratios that match U.S. data.

We model catastrophes as Poisson events with some mean arrival rate, and an impact characterized by a one-parameter power probability distribution. Thus the characteristics of catastrophes are captured by two parameters. Leaving these two parameters unconstrained, we calibrate our model so that it fits the basic data for the consumption-investment ratio, the risk-free interest rate, the equity premium, Tobin’s q , and the average real growth rate. We thereby calculate the implied characteristics of catastrophes, and also determine how those characteristics vary over a range of values for the preference parameters.³

We show that the mean annual arrival rate is less than .04, but cannot pin it down more precisely than that. This is not surprising, given that catastrophes are indeed “rare” events. We find that conditional on the value for the index of risk aversion, the expected loss from a catastrophic event is in the range of about 15 to 30 percent of the capital stock.

Our results also yield information about the values of behavioral parameters. In particular, we provide evidence that the elasticity of intertemporal substitution is below 0.5, and that expected CRRA utility may be a reasonable approximation for modeling preferences.

We use our calibrated model to address the third question that we raised at the outset: How much should society be willing to pay to reduce the probability or likely impact of a catastrophic event? We calculate a tax-based measure of “willingness to pay” (WTP). In

³A more general version of the model would allow the impact of a catastrophe to be temporary by assuming that following an initial drop, productivity mean-reverts to its original level. This would introduce a third parameter to the characterization of catastrophes — the rate of mean reversion. The three catastrophe parameters could still be calculated as an output of the model’s calibration.

our model a permanent tax on consumption is non-distortionary, equivalent to a lump-sum tax, and equivalent to a reduction in the current capital stock by an amount equal to the tax rate. Thus our WTP is the permanent percentage tax rate that society should be willing to accept to reduce the mean arrival rate of a catastrophic event from its calibrated value to some fraction of that value. This approach allows us to avoid estimating the cost of reducing the mean arrival rate, which is presumably a convex function of the size of the reduction.

The questions we address have been the focus of a growing literature, the roots of which go back to the observation by Rietz (1988) that low-probability catastrophes could, in theory at least, explain the equity premium puzzle, i.e., could help reconcile a relatively large equity premium (5 to 7%) and low real risk-free rate of interest (1 to 2%) with a moderate degree of risk aversion on the part of households. Rietz’s article received little attention until the recent work of Barro (2006, 2009) and Weitzman (2007). Barro (2006) assembled data on “consumption disasters,” defined as reductions in real GDP of 15% or more, for a panel of 35 countries over the past century. He estimated the Poisson arrival rate of such events (just under 2% per year) and the distribution of the drop in GDP. Using a pure exchange (fruit tree) model of the economy similar to that of Rietz, Barro showed that these numbers are roughly consistent with the observed equity premium and real risk-free rate in the U.S.⁴

Barro (2009) extended his earlier work by generalizing the model to include an *AK* production technology and Epstein-Weil-Zin (EWZ) recursive preferences, thereby endogenizing savings and investment, and disentangling the index of risk aversion from the elasticity of intertemporal substitution. Using his earlier estimates of the mean arrival rate and impact distribution, the model could again match the observed equity premium and risk-free rate. The model also implied large welfare costs of disasters: depending on parameter values, equivalent to a 5 to 25% reduction in the initial capital stock and GDP.⁵

While Barro’s (2009) *AK* production model can explain the equity premium and risk-

⁴Concurrently, Weitzman (2007) showed that the equity premium and real risk-free rate puzzles could be explained by “structural uncertainty” in which one or more key parameters, such as the true variance of equity returns, is estimated through Bayesian updating.

⁵We use the word “model,” but Barro (2009) has three different models: a pure exchange economy with recursive preferences, a model with endogenous labor supply, and the *AK* model that we discuss here.

free rate, its calibration is inconsistent with other basic economic variables. For example, it predicts a consumption-investment ratio of about 1:3, instead of matching the roughly 3:1 ratio in the data. Also, because consumption and investment goods are freely interchangeable in his model, Tobin's q (marginal and average) always equals one, which is unrealistic.

Several authors have extended Barro's work. Gourio (2008), for example, used an exchange economy model with recursive preferences but allowed consumption disasters to have limited duration. He found that the effect of recoveries on the equity premium could be positive or negative, depending on the elasticity of intertemporal substitution. Gabaix (2008) and Wachter (2008) showed that a time-varying Poisson arrival rate could explain the high volatility of the stock market (in addition to the equity premium and real risk-free rate).

Other studies have sought improved estimates of the event arrival rate and impact. For example, Barro and Ursúa (2008) exploit an extended dataset based on consumption instead of GDP, and Barro, Nakamura, Steinsson, and Ursúa (2009) estimate a more general model that accounts for recoveries. While these studies provide a better understanding of the characteristics of historical "consumption disasters," they are limited in two respects. First, many of the included disasters are manifestations of three global events — the two World Wars and the Great Depression. Second, the possible catastrophic events that we think are of greatest interest today have little or no historical precedent — there is no data, for example, on the frequency or impact of nuclear or biological terrorist attacks.

Consider the forty-year period beginning around 1950 and ending with the breakup of the Soviet Union. During that time there was one potential catastrophic event that dominated all others: the possibility of an all-out nuclear war between the U.S. and the Soviet Union. The fear of such of an event was based partly on the possibility of a mistake: One side might see something threatening on a radar screen, and, unable to get sufficient reassurance from a phone call, launch its own missiles. What was the mean arrival rate for such an event and the probability distribution for its impact? Although the Department of Defense, the RAND Corporation, and others did studies to address these questions, there was no historical precedent on which to base estimates. Thus we take a very different approach and ask what event arrival rate and impact distribution are implied by basic economic data.

In the next section we lay out a general equilibrium model that incorporates catastrophic shocks, discuss its solution, and explain how its calibration yields information about shocks. Section 3 shows the calibration results, and discusses the implications for the nature of catastrophic shocks, and for household preferences. Section 4 discusses our application of the model to policy analysis, and in particular, the calculation of WTP. Section 5 concludes.

2 Framework.

We need a tractable general equilibrium model that describes production, capital accumulation, and household preferences, and that includes the possible arrival of catastrophic events. Also, the calibrated model should match basic economic aggregates: the consumption-investment ratio, real risk-free rate, equity premium, Tobin's q , and real growth rate.

We construct a general equilibrium continuous-time model in which: (i) a representative consumer has recursive preferences; (ii) output is given by an AK technology; (iii) investment involves a generalized adjustment cost that reflects the expense and time required to install capital, so that $q \neq 1$; (iv) catastrophic shocks occur as Poisson arrivals resulting in the loss of a random fraction of the capital stock. Despite its generality, the model yields closed-form solutions for equilibrium allocations and pricing.

2.1 Building Blocks.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of EWZ preferences, so that a representative consumer has homothetic recursive preferences given by:

$$V_t = \mathcal{E}_t \left[\int_t^\infty f(C_s, V_s) ds \right] , \quad (1)$$

where

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1 - \gamma)V)^\omega}{((1 - \gamma)V)^{\omega-1}} . \quad (2)$$

Here $\rho > 0$ is the rate of time preference, ψ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and we define the parameter ω as:

$$\omega = \frac{1 - \psi^{-1}}{1 - \gamma} . \quad (3)$$

Unlike standard time-additive separable isoelastic utility specifications, this recursive preference allows us to separate risk aversion from the elasticity of intertemporal substitution.⁶

Note that if $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by additively separable aggregator:

$$f(C, V) = \frac{\rho C^{1-\gamma}}{1-\gamma} - \rho V. \quad (4)$$

One of the questions we address is whether ω is in fact close to 1, so that the simple CRRA utility function is a reasonable approximation for modeling purposes. In particular, we examine how equilibrium allocation and pricing constrains the model's parameters, including the elasticity of intertemporal substitution and index of risk aversion.

Production. Aggregate output has an AK production technology:

$$Y = AK, \quad (5)$$

where A is a constant. One can interpret K as the stock of physical capital as traditionally measured, so that A is a coefficient that incorporates the effects of labor and materials on production.⁷ Note that A is an output of the model's calibration.

Catastrophic Shocks. We model catastrophic shocks as Poisson arrivals with mean arrival rate λ . A shock destroys a stochastic fraction $(1 - Z)$ of the capital stock K , where the remaining capital stock, Z , is a well-behaved random variable with $0 \leq Z \leq 1$. By well-behaved, we mean that the moments $\mathcal{E}(Z^n)$ exist, where $n = 1, 1 - \gamma$, and $-\gamma$.

Investment and Capital Accumulation. Letting I denote aggregate investment, the capital stock K evolves as:

$$dK(t) = \Phi(I(t), K(t))dt + \sigma K(t)dW(t) - (1 - Z)K(t)dJ(t). \quad (6)$$

⁶Note from eqn. (2) that the marginal benefit of consumption is $f_C = \rho C^{-\psi-1}/[(1-\gamma)V]^{\omega-1}$, and thus depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

⁷For example, if capital and labor are used in fixed proportions, $Y = A'K^aL^{1-a}$ and $L = lK$, so that $Y = AK$ with $A = A'l^{1-a}$. We might then expect A to be around 0.2 to 0.3, corresponding to rough measures of the output-capital ratio. An alternative interpretation of K is a composite capital stock that includes all forms of capital — plant and equipment as well as human and infrastructure capital — which would imply smaller values for A .

Here the volatility parameter σ captures “normal” volatility, and $J(t)$ is a pure jump process with mean arrival rate λ that captures catastrophic events; if a jump occurs, K falls by the random fraction $(1 - Z)$. The adjustment cost function $\Phi(I, K)$ captures the effects of depreciation and the costs of installing capital and making it productive. We assume $\Phi(I, K)$ is homogeneous of degree one in I and K and thus can be written as:

$$\Phi(I, K) = \phi(i)K , \quad (7)$$

where $i = I/K$ and $\phi(i)$ is increasing and concave. As we will see, the solution (and thus calibration) of the model use only the level of $\phi(i^*)$ and its slope $\phi'(i^*)$ at the equilibrium investment-capital ratio i^* , so we do not require any particular specification for $\phi(i)$.

Competitive Equilibrium. We have two more conditions: (1) Investment always equals saving, i.e., $I(t) = Y(t) - C(t)$ at all $t \geq 0$. (2) Financial markets clear, i.e., there is a zero net supply of bonds, and total wealth is all marketable equity and thus equals the market value of the capital stock $Q(t)$. With these conditions, we can solve the social planner’s problem to obtain the competitive equilibrium.

The Bellman equation for the social planner’s problem is:

$$0 = \max_C \left\{ f(C, V) + \Phi(I, K)V'(K) + \frac{1}{2}\sigma^2 K^2 V''(K) + \lambda \mathcal{E} [V(ZK) - V(K)] \right\} . \quad (8)$$

Using the identity $C + I = Y$, we have the following first-order condition for I :

$$f_C(C, V) = \phi'(i)V'(K) . \quad (9)$$

The left-hand side of eqn. (9) is the marginal benefit of consumption and the right-hand side is its marginal cost, which equals the marginal value of capital $V'(K)$ times the marginal efficiency of converting a unit of the consumption good into a unit of capital, $\phi'(i)$.

2.2 Model Solution.

Exploiting the homogeneity property of the value function, we conjecture and later confirm that the value function takes the following form:

$$V(K) = \frac{1}{1 - \gamma} (bK)^{1 - \gamma} , \quad (10)$$

where b is a coefficient determined as part of the solution. Let $c = C/K = A - i$. (Lower-case letters in this paper express quantities relative to the capital stock K .) The Appendix shows that b is related to the equilibrium level of investment by:

$$b = (A - i^*)^{1/(1-\psi)} \left(\frac{\rho}{\phi'(i^*)} \right)^{-\psi/(1-\psi)}, \quad (11)$$

where the equilibrium investment-capital ratio i^* solves the following implicit equation:

$$A - i = \frac{1}{\phi'(i)} \left[\rho + (\psi^{-1} - 1) \left(\phi(i) - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma} \mathcal{E} (1 - Z^{1-\gamma}) \right) \right]. \quad (12)$$

Equilibrium capital accumulation is then given by

$$dK(t)/K(t) = \phi(i^*)dt + \sigma dW(t) - (1 - Z)dJ(t), \quad (13)$$

where i^* is the solution of eqn. (12). Note that

$$g = \phi(i^*) \quad (14)$$

is the expected growth rate (conditional on no catastrophic shocks).

Using the solution sketched out above, we show in the Appendix that the model can be summed up by five key equations, which can be interpreted as the decentralized competitive-market implementation of the social planner's solution:

$$i = A - c \quad (15)$$

$$q = \frac{1}{\phi'(i)} \quad (16)$$

$$c = \left[\rho + (\psi^{-1} - 1) \left(g - \frac{\gamma\sigma^2}{2} - \frac{\lambda}{1-\gamma} \mathcal{E} (1 - Z^{1-\gamma}) \right) \right] q \quad (17)$$

$$r = \rho + \psi^{-1}g - \frac{\gamma(\psi^{-1} + 1)\sigma^2}{2} - \lambda\mathcal{E} \left[(\psi^{-1} - \gamma) \left(\frac{1 - Z^{1-\gamma}}{1 - \gamma} \right) + (Z^{-\gamma} - 1) \right] \quad (18)$$

$$rp = \gamma\sigma^2 + \lambda\mathcal{E} [(1 - Z) (Z^{-\gamma} - 1)] \quad (19)$$

Eqn. (15) is simply an accounting identity that equates saving and investment. Eqn. (16) is the first-order condition for producers. Re-writing it as $\phi'(i)q = 1$, it equates the marginal

benefit of an extra unit of investment (which at the margin yields $\phi'(i)$ units of capital, each of which is worth q) with its marginal opportunity cost (1 unit of the consumption good).

Eqn. (17) is the first-order condition for consumers. It equates consumption (normalized by the capital stock) to the marginal propensity to consume (MPC) out of wealth (everything in the square brackets) times q , which is the value of a marginal unit of wealth. Note that the entire capital stock is marketable and its value is qK . What drives the MPC, c/q ? Looking inside the square brackets, if $\psi = 1$, wealth and substitution effects just offset each other, and the MPC = c/q is equal to the rate of time preference ρ . More generally, if $\psi^{-1} < (>)$ 1, the MPC $< (>)$ ρ by an amount proportional to the quantity in the large parentheses, which can be interpreted as a risk-adjusted growth rate:

$$\hat{g} \equiv g - \frac{1}{2}\gamma\sigma^2 - \frac{\lambda}{1-\gamma}\mathcal{E}\left(1 - Z^{1-\gamma}\right) . \quad (20)$$

With no stochastic changes in K , the growth rate is $g = \phi(i)$; that rate is reduced in risk-adjusted terms by “normal” fluctuations in K (the second term) and by the prospect of downward jumps in K (the last term). Note that \hat{g} differs from the simple expected growth rate accounting for jumps, $\bar{g} = g - \lambda\mathcal{E}(1 - Z)$, in that \bar{g} only accounts for the expected change in the growth rate due to possible jumps without accounting for risk aversion.

Eqn. (18) for the interest rate r may be interpreted as a generalized Ramsey rule. If $\psi^{-1} = \gamma$ so that preferences simplify to CRRA expected utility, and if there were no stochastic changes in K , the deterministic Ramsey rule $r = \rho + \gamma g$ would hold. More generally, the third term captures the precautionary savings effect under recursive preferences of continuous stochastic fluctuations in K , and the last term adjusts for downward jumps in K .

Finally, eqn. (19) describes the equity risk premium, rp . The first term on the RHS is the usual risk premium in diffusion models (see, e.g., Breeden (1979) and Lucas (1978)), and the second term is the increase in the premium that is due to jumps in K . When a jump occurs, $(1 - Z)$ is the fraction of loss, and $(Z^{-\gamma} - 1)$ is the percentage increase in marginal utility from that loss, i.e., the price of risk. The jump component of the equity risk premium is given by λ times the expectation of the product of these two random variables. Note that the fraction of losses and the increase of marginal utility are positively correlated. This positive

correlation substantially contributes to the risk premium. For example, in the limiting case where the loss is close to 100%, the increase in marginal utility is effectively infinite.

2.3 The Distribution for Shocks.

The solution of the model presented above applies to any well-behaved distribution for Z . We assume, however, that Z follows a power distribution over $(0,1)$ with parameter $\alpha > 0$:

$$f_Z(z) = \alpha z^{\alpha-1} \quad ; \quad 0 \leq z \leq 1 \quad , \quad (21)$$

so that $\Pr(Z \geq z) = 1 - z^\alpha$, and $\mathcal{E}(Z) = \alpha/(\alpha + 1)$. Thus a large value of α implies a relatively small expected loss $\mathcal{E}(1 - Z)$.

Note that the distribution given by eqn. (21) is quite general. If $\alpha = 1$, Z follows a uniform distribution. For any $\alpha > 0$, eqn. (21) implies that $-\ln Z$ is exponentially distributed with mean $\mathcal{E}(-\ln Z) = 1/\alpha$. eqn. (21) also implies that the inverse of the remaining fraction of the capital stock, $M = 1/Z$, follows a Pareto distribution given by

$$f_M(m) = \alpha m^{-\alpha-1} \quad , \quad m > 1 \quad .$$

The power and Pareto distributions are fat-tailed and often used to model random events.

The solution of the model is simplified by making use of the power distribution for Z . We need three moments of Z , namely $\mathcal{E}(Z^n)$ where $n = 1, 1 - \gamma$, and $-\gamma$. From eqn. (21),

$$\mathcal{E}(Z^n) = \alpha/(\alpha + n) \quad , \quad (22)$$

provided that $\alpha + n > 0$. Since the smallest relevant value of n is $-\gamma$, we require $\alpha > \gamma$. Thus $\mathcal{E}(1 - Z) = 1/(\alpha + 1)$ is the expected loss if an event occurs, and $\mathcal{E}(Z^{-\gamma} - 1) = \gamma/(\alpha - \gamma)$ is the expected percentage increase in marginal utility from the loss; both are decreasing in α .

Using eqn. (22), we can rewrite eqns. (17), (18), and (19) as:

$$\lambda = (\alpha + 1) \left[\frac{c}{q} - r - rp + g \right] \quad (23)$$

$$r = \rho + \psi^{-1}g - \frac{\gamma(\psi^{-1} + 1)\sigma^2}{2} - \lambda \left[\frac{(\psi^{-1} - \gamma)(\alpha - \gamma) + \gamma(\alpha - \gamma + 1)}{(\alpha - \gamma)(\alpha - \gamma + 1)} \right] \quad (24)$$

$$rp = \gamma\sigma^2 + \lambda\gamma \left[\frac{1}{\alpha - \gamma} - \frac{\alpha}{(\alpha + 1)(\alpha + 1 - \gamma)} \right] \quad (25)$$

We can now use the model to back out the implied values of λ and α , explore the admissible range of values for ψ , and evaluate the constraint of expected CRRA utility.

2.4 Calibration Procedure.

We first calibrate the model using “average” or “consensus” values for the following variables or parameters based on U.S. data: the consumption-investment ratio c/i , marginal (and average) q , the real risk-free interest rate r , the equity risk premium rp , the average “normal” growth rate g (i.e., without accounting for disasters), and the normal volatility σ . We also specify values for the index of risk aversion γ and the rate of time preference ρ .

This leaves six unknowns: the economic variables c , i and A ; the elasticity of intertemporal substitution ψ , and the two parameters describing the characteristics of catastrophic shocks, λ and α . We have the five equations (15), (16), and (23) to (25), so the set of unknowns is under-identified. We therefore calibrate the model for different values for A , and consider solutions over the range of admissible values for A , i.e., values for which the resulting $\lambda > 0$ and $\psi > 0$. In fact, this limits A to a fairly narrow range (and thus roughly identifies A). We can also calibrate the model imposing the constraint of expected CRRA utility, i.e., $\psi = 1/\gamma$, in which case the five remaining unknowns are exactly identified.

We also calculate the implied value for the marginal propensity to consume out of wealth, $MPC = c/q$. Using the accounting identity $i = A - c$,

$$MPC = \frac{c}{q} = \left[\frac{c/i}{1 + (c/i)} \right] \frac{A}{q}. \quad (26)$$

Given admissible values for A , we expect c/q to be in the range of .05 to .07. This is a further check on the model’s calibration.

3 Results.

Our objective is to calculate λ , α , and the elasticity of intertemporal substitution ψ as calibration outputs, and see how these parameters vary as we change key inputs. We present

Table 1: Baseline Calibrations

Note: $r = .02$, $c/i = 3$, $rp = .06$, $q = 1.5$, $\rho = .02$, $\sigma = .02$, $g = .025$; Boldface entries correspond to expected CRRA utility.					
$\gamma = 2$					
A	MPC	ψ	λ	α	$\mathcal{E}(1 - Z)$
0.112	.056	0.3769	0.0031	2.0983	0.3228
0.116	.058	0.3081	0.0098	2.2818	0.3047
0.120	.060	0.2413	0.0173	2.454	0.2895
0.124	.062	0.1757	0.0253	2.619	0.2763
0.128	.064	0.1108	0.034	2.779	0.2646
$\gamma = 4$					
0.112	.056	0.3584	0.0053	4.2956	0.1888
0.116	.058	0.2746	0.0173	4.761	0.1736
0.1173	.059	0.25	0.0214	4.8929	0.1697
0.120	.060	0.1991	0.0308	5.1605	0.1623
0.124	.062	0.1272	0.0457	5.5279	0.1532

a set of calibration results, and discuss their implications for the values of λ , α and ψ . We then turn to the role of adjustment costs and compare our results with those of Barro (2009), in which λ and the distribution for Z were exogenous. This is useful because it illustrates the importance of adjustment costs, and the implications of certain parameter choices.

3.1 Baseline Calibrations.

We calibrate the model leaving four parameters unconstrained: the mean arrival rate of catastrophes λ , the impact distribution parameter α , the productivity parameter A , and the elasticity of intertemporal substitution ψ . We constrain the consumption-investment ratio $c/i = 3$, the annual real risk-free interest rate $r = .02$, the equity risk premium $rp = .06$, marginal (and average) $q = 1.5$, the average “normal” (i.e., absent a catastrophe) growth rate $g = .025$, and the normal volatility $\sigma = .02$. We use two alternative values for the index of risk aversion γ , 2 and 4, and set the annual rate of time preference $\rho = .02$.

The results are summarized in Table 1. For each value of γ , the table shows the admissible range of A , i.e., values of A for which the resulting values of ψ and λ are positive. This range

is narrow; about .11 to .13. For each A , the table also shows the corresponding MPC, the resulting values of ψ , λ , α , and the expected loss $\mathcal{E}(1 - Z)$. (Note from eqn. (22) that α and $\mathcal{E}(1 - Z)$ are directly related.) For either value of γ , the MPC is around .06, well within the consensus range. Likewise, ψ is always in the range of about 0.1 to 0.4.

Several conclusions can be drawn from these results. First, we obtain values of ψ that are always below 0.5. Estimates of ψ in the literature vary considerably, ranging from those in the table to values as high as 2.⁸ But as our calibrations show, basic macro data are inconsistent with values of ψ much above 0.5. In fact, as can be seen from eqns. (17) and (20), if one's priors are that the risk-adjusted growth rate \hat{g} is positive and the MPC = c/q is larger than the rate of time preference ρ , then ψ must be less than 1.

Second, these results are consistent with the view that restricting preferences to expected CRRA utility is not a bad approximation for modeling purposes. When $\gamma = 2$, expected utility implies that $\psi = 0.5$, and although our estimates of ψ are below 0.5, they are not far below. When $\gamma = 4$, expected utility implies $\psi = 0.25$, which is in the middle of the range of our estimates for ψ , and is shown as the row of boldface numbers in Table 1.

Third, we find that λ is less than about .04, so that catastrophes are indeed rare events, but we cannot estimate its value more precisely than that. In fact, we cannot rule out values of λ close to zero. In a sense, our estimates of λ are similar to our estimates of ψ ; we can only provide a range, .1 to .4 in the case of ψ and 0 to .04 in the case of λ . As we will show later, however, this range for λ , combined with the estimated impact distribution, has strong policy implications. On the other hand, the estimates of α and the expected loss $\mathcal{E}(1 - Z)$ depend on γ , but given γ , the loss distribution is reasonably well identified. If, for example, one had a strong prior that $\gamma = 2$, then the expected loss should a catastrophic shock occur is in the range of 26 to 32 percent.

⁸Bansal and Yaron (2004) argue that the elasticity of intertemporal substitution is above unity and use 1.5 in their long-run risk model. Attanasio and Vissing-Jorgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. Using micro and macro evidence, Guvenen (2006) attempts to reconcile the different estimates. He argues that the elasticity depends on wealth, which is much less evenly distributed than consumption, so that estimates based on aggregate consumption uncover the low of the majority of the population. The Appendix to Hall (2009) provides a brief survey of estimates in the literature.

In summary, given a prior on γ , the model provides a good fix on the expected loss $\mathcal{E}(1 - Z)$, but only a range (of 0 to .04) for the mean arrival rate λ . Thus when we use the model for policy analysis (see below), we must do so for different values of A and hence λ .

3.2 The Role of Adjustment Costs.

How important are adjustment costs for a model like ours? To address this question and do welfare calculations, we must specify an adjustment cost function $\phi(i)$. We use a quadratic function, which can be viewed as a second-order approximation to a more general one:

$$\phi(i) = i - \frac{1}{2}\theta i^2 - \delta . \tag{27}$$

In Table 1, when $\gamma = 2$ the calibrated values of θ range from 10.4 to 12.1 ($\theta = 11.1$ when $A = .120$). When $\gamma = 4$, the values range from 10.7 to 12.1 ($\theta = 11.4$ when $A = .1173$).

One way to explore the role of adjustment costs is to review Barro’s (2009) calibration results, but amend his model to include adjustment costs. Based on historic “consumption disasters” for a large panel of countries, Barro estimated the mean arrival rate λ to be .017 (roughly in the middle of our estimated range for λ). He set $\gamma = 4$, and based on an empirical distribution for consumption declines, he estimated the three moments $\mathcal{E}(Z)$, $\mathcal{E}(Z^{1-\gamma})$, and $\mathcal{E}(Z^{-\gamma})$. He also chose the following parameter values: $\psi = 2$, $\rho = .052$, $\sigma = .02$, and $A = .174$. As we noted earlier, economists differ in their views about ψ , but a value of 2 is at the high end of the range of numbers that have appeared in the literature. Likewise, a more typical value for ρ , the rate of time preference, would be closer to .02.

The first row of the top panel of Table 2 shows this calibration of Barro’s model; there are no adjustment costs so $q = 1$. The model gives a sensible estimate of the risk-free rate r , but yields a consumption-investment ratio of only 0.38, whereas the actual ratio is about 3 or 4. The table shows how the results change as the adjustment cost parameter θ in eqn. (27) is increased so that q increases. As we increase θ , i falls and c increases. When $\theta = 8$, c/i roughly matches the actual data, and q is around 1.4, also a rough match to the data. However, the real risk-free rate falls below -3% , the growth rate falls to around -8% , and the MPC is around 10%. The basic problem is that given the values of the preference

Table 2: Effects of Adjustment Costs

(1) Barro (2009) Parameters: $\gamma = 4$, $\psi = 2$, $\rho = .052$, $\sigma = .02$, $A = .174$, $\lambda = .017$, $\mathcal{E}(Z) = .71$, $\mathcal{E}(Z^{1-\gamma}) = 4.05$, $\mathcal{E}(Z^{-\gamma}) = 7.69$							
θ	i	c	c/i	r	q	g	MPC
0	0.126	0.048	0.381	.011	1.000	.025	.048
4	0.062	0.112	1.806	-.025	1.333	-.046	.084
8	0.038	0.136	3.579	-.036	1.434	-.068	.095
12	0.027	0.147	5.512	-.041	1.472	-.078	.100
20	0.017	0.157	9.234	-.045	1.510	-.086	.104
(2) Our Parameters: $\gamma = 4$, $\psi = 0.25$, $\rho = .02$, $\sigma = .02$, $A = .117$, $\lambda = .021$, $\mathcal{E}(Z) = .830$, $\mathcal{E}(Z^{1-\gamma}) = 2.585$, $\mathcal{E}(Z^{-\gamma}) = 5.479$							
θ	i	c	c/i	r	q	g	MPC
0	0.033	0.084	2.56	.040	1.000	.030	.084
4	0.032	0.086	2.69	.033	1.146	.028	.075
8	0.030	0.087	2.85	.026	1.322	.026	.066
11.37	0.029	0.088	3.000	0.020	1.500	.025	.059
20	0.026	0.091	3.48	.005	2.101	.021	.043

parameters (particularly ψ and ρ) in the top panel of the table, along with the exogenous inputs for λ and the moments of Z , the model simply cannot match all of the basic economic facts, even allowing for adjustment costs.

The bottom panel of Table 2 shows the calibration corresponding to the middle (boldface) row of the bottom panel of Table 1, i.e., expected utility with $\gamma = 4$ and $\sigma = .02$ as in the top panel, but the other parameter values are those we used as inputs to our calibration or were calibration outputs ($\psi = 0.25$, $\rho = .02$, $A = .117$, $\lambda = .021$, $\mathcal{E}(Z) = .830$, $\mathcal{E}(Z^{1-\gamma}) = 2.585$, and $\mathcal{E}(Z^{-\gamma}) = 5.479$). We again vary θ . (The boldface row corresponds to our calibrated value of θ , 11.37.) Note that if we set $\theta = 0$, c/i becomes too small, and r and g become too large. As θ increases, the cost of investing increases, so consumption increases, investment falls, and both r and g fall. (Also, q increases with θ because with higher adjustment costs, installed capital earns greater rents over newly purchased capital.)

3.3 Alternative Inputs.

Our basic calibrations used what we called “average” or “consensus” values for various parameters and economic variables. One might argue that the values of ρ , r , etc., should be somewhat different from those we used. We therefore re-calibrated the model making changes in each of several of these inputs. The results are shown in Table 3. The first row in each panel of the table shows the base case corresponding to the inputs we used in the calibrations shown in Table 1. Holding everything else constant in each case, we then increased ρ from .02 to .04, decreased r from .02 to 0, decreased the risk premium rp from .06 to .04, increased c/i from 3 to 4, and increased q from 1.5 to 2. Each of these new calibrations yields a range of admissible values for A ; in Table 3 we report the mid-point of that admissible range and the corresponding values of ψ , λ , and $\mathcal{E}(1 - Z)$.

As the table shows, our basic results are not highly sensitive to changes in these input values. In all cases, our mid-range estimates of ψ are between .24 and .40, and our estimates of λ are between .015 and .035. As before, the estimates of the expected loss depend on the value of γ , but vary only slightly with changes in ρ , r , etc. Also, note that increasing ρ from .02 to .04 and decreasing r from .02 to 0 have exactly the same effect on ψ , λ , and $\mathcal{E}(1 - Z)$. As can be seen from eqns. (17) and (23) to (25), holding c/i , g , and rp constant, both c/q and r respond one-to-one to changes in ρ , so a 1-percentage point increase in ρ has the same effect as a 1-percentage point decrease in r . Likewise, a 33-percent increase in c/i from 3 to 4 has the same effect on ψ , λ , and $\mathcal{E}(1 - Z)$ as a 33-percent increase in q from 1.5 to 2.

4 Policy Implications.

We now turn to the last question raised in the first paragraph of this paper: What is society’s willingness to pay to reduce the probability or likely impact of catastrophic events? Our measure of WTP is the maximum *permanent* consumption tax rate τ that society would be willing to accept if the resulting stream of government revenue could finance whatever activities would permanently reduce the mean arrival rate of a catastrophe. We also consider a tax that would reduce the average impact of a catastrophe by increasing α , and thereby

Table 3: Changes in Input Values

Note: Unless otherwise indicated, $r = .02$, $c/i = 3$, $rp = .06$, $\rho = .02$, $\sigma = .02$, $g = .025$, $q = 1.5$. All entries are for mid-point of admissible range of A .				
$\gamma = 2$				
Change	A	ψ	λ	$\mathcal{E}(1 - Z)$
BASE CASE	0.119	0.2546	.0157	.2924
$\rho = .04$	0.121	0.3766	.0182	.2878
$r = 0$	0.081	0.3766	.0182	.2878
$rp = .04$	0.082	0.3655	.0223	.2651
$c/i = 4$	0.112	0.2537	.0158	.2922
$q = 2$	0.159	0.2537	.0158	.2922
$\gamma = 4$				
BASE CASE	0.118	0.2438	.0225	.1688
$\rho = .04$	0.118	0.3855	.0242	.1673
$r = 0$	0.078	0.3855	.0242	.1673
$rp = .04$	0.080	0.3580	.0348	.1495
$c/i = 4$	0.110	0.2428	.0227	.1686
$q = 2$	0.157	0.2424	.0228	.1686

reducing the expected loss, as well as the variance of the loss.

What is the effect of a permanent consumption tax? Given investment $I(t)$ and output $Y(t)$, households pay $\tau(Y - I)$ to the government and consume the remainder:

$$C(t) = (1 - \tau)[Y(t) - I(t)] . \quad (28)$$

Therefore the first-order condition of eqn. (9) becomes

$$(1 - \tau)f_C(C, V) = \phi'(i)V'(K) . \quad (29)$$

A consumption tax has two opposing effects. First, it lowers the marginal cost of current investment (because the part of output allocated to investment is not taxed), making investment more attractive, all else equal. Second, because the tax is permanent, it lowers the marginal benefit of current investment because any future dividends generated from this investment (i.e., net output available for consumption) will also be taxed, making invest-

ment less attractive. As shown below and in the Appendix, in equilibrium, these two effects exactly offset each other, leaving the investment decision unaffected.

Now, consider a tax to reduce λ . Let $V(K; \lambda, \tau)$ denote the value function for given values of λ and τ . We conjecture that V takes the homothetic form:

$$V(K; \lambda, \tau) = \frac{1}{1 - \gamma} (b(\lambda; \tau)K)^{1-\gamma}, \quad (30)$$

Intuitively, $b(\lambda; \tau)$ measures certainty-equivalent wealth (per unit of capital) when consumption is permanently taxed at rate τ . Let $V^*(K; \lambda)$ and b^* denote the corresponding quantities in the absence of a tax as in Section 2, i.e., $V^*(K; \lambda) = V(K; \lambda, 0)$ and $b^*(\lambda) = b(\lambda; 0)$. In the Appendix we show that with a tax,

$$b(\lambda; \tau) = (1 - \tau)b^*(\lambda). \quad (31)$$

Note that $b^*(\lambda)$ in eqn. (11) is evaluated at the equilibrium i^* (without a tax). In this case, the solution to eqn. (12) remains the same, and the investment-capital ratio i is unchanged. Furthermore, the permanent consumption tax acts as a lump-sum tax and is equivalent in terms of welfare to an initial one-time percentage reduction of the capital stock by the amount of the tax rate, τ . The tax induces no distortion in terms of economic decisions. Total output Y , investment I , and the evolution of K remain the same; the only change is in the allocation of output between consumers and the government.

The intuition here is that because the tax lowers consumption by the same fraction every period, it leaves households' intertemporal marginal rate of substitution unchanged. (Recall that in our model preferences are homothetic and the economy follows a stochastic balanced growth path.) Thus holding λ fixed, the equilibrium investment and growth paths are the same with or without the tax. A permanent consumption tax in our model is therefore non-distortionary; there is no incentive to change intertemporal investment decisions in response to the tax. Household consumption each period is simply reduced by the fraction τ .

4.1 Willingness to Pay.

How large a tax rate would society accept to reduce λ to λ' ? Households are indifferent between (a) no tax and a likelihood of catastrophe λ and (b) paying a permanent tax at rate

τ to reduce the likelihood to λ' if the following condition holds:

$$V(K; \lambda', \tau) = V(K; \lambda, 0) . \quad (32)$$

This implies:

$$b(\lambda'; \tau) = (1 - \tau)b^*(\lambda') = b^*(\lambda) , \quad (33)$$

where the first equality follows from equations (30) and (31), and the second is the indifference condition between (a) and (b).

Thus to reduce the likelihood of a catastrophe from λ to λ' , households would be willing to pay a consumption tax at the following constant rate:

$$\tau(\lambda, \lambda') = 1 - \frac{b^*(\lambda)}{b^*(\lambda')} . \quad (34)$$

As can be seen from eqns. (30), (31), and (34), from the household's perspective a permanent tax at rate τ is equivalent to giving up a fraction τ of the capital stock. This is not surprising because the tax does not affect investment or (because the equilibrium MRS is also unaffected) the equilibrium discount rate, so that the total value of capital is unchanged; a fraction τ of ownership is simply transferred from households to the government.

For a given value of λ , market valuation of the aggregate capital stock is the same with or without a tax because neither the cash flows nor the discount rate change. Therefore, the effect of the tax on the stock market is just the effect of reducing λ , i.e., the market value of a unit of capital changes by the fraction:

$$\frac{q(\lambda')}{q(\lambda)} = \frac{\phi'(i^*(\lambda))}{\phi'(i^*(\lambda'))} . \quad (35)$$

Note that the optimal investment-capital ratio i^* is a function of λ .

We focused on a tax used to reduce λ , but the results also apply if the tax is used to reduce the expected impact of a catastrophic event. By substituting α in place of λ , eqn. (34) can be used to find the maximum tax rate households would accept to increase the impact distribution parameter from α to $\alpha' > \alpha$. Note from eqn. (22) that increasing α reduces the expected loss $\mathcal{E}(1 - Z)$ as well as the variance (and higher-order moments) of the loss.

Table 4: WTP Calculations

Note: $r = .02$, $c/i = 3$, $rp = .06$, $\rho = .02$, $\sigma = .02$, $g = .025$, $q = 1.5$						
$\gamma = 2$						
			$\lambda'/\lambda = .8$	$\lambda'/\lambda = .5$	$\lambda'/\lambda = 0$	$\frac{\mathcal{E}(1-Z')}{\mathcal{E}(1-Z)} = .5$
A	λ	$\mathcal{E}(1-Z)$	τ^*			
0.112	.0031	0.3228	.01	.02	.05	.04
0.120	.0173	0.2895	.04	.09	.15	.12
0.128	.0340	0.2646	.06	.12	.21	.16
$\gamma = 4$						
0.112	.0053	0.1888	.01	.04	.07	.06
0.117	.0214	0.1697	.04	.09	.16	.13
0.124	.0457	0.1532	.05	.12	.21	.16

4.2 Tax Calculations.

Table 4 shows the maximum permanent tax rate society would accept to reduce λ by 20 percent (i.e., $\lambda'/\lambda = .8$), 50, and 100 percent for different values of A from the calibrations in Table 1. We also show the corresponding values of λ and $\mathcal{E}(1-Z)$. Not surprisingly, these tax rates vary depending on the base value of λ . However, even for low base values (the top row of each panel), a significant permanent tax (2 to 7 percent) would be warranted if it could reduce λ in half or eliminate the risk entirely. And the same holds for a tax that would leave λ unchanged, but would reduce the expected loss from an event by 50 percent.⁹

These results have strong policy implications. Although we can not pin down λ precisely, our results suggest that a reasonable estimate of the annual risk of a catastrophe is around .02, which is roughly in the middle of our calibrated range. A tax as high as 15% would then be justified if the resulting revenues could support activities that would significantly reduce that risk. For example, for either value of γ , if λ is around .017 to .02, a 50-percent reduction in λ would justify a permanent consumption tax of about 9 percent.

⁹Note, however, that a 50% reduction in the expected loss also implies a substantial reduction in the variance of the loss. For example, if $\gamma = 2$ and $\lambda = .0173$, the 12-percent consumption tax that cuts $\mathcal{E}(1-Z)$ in half does so by increasing α from 2.45 to 5.91, which reduces the variance of $(1-Z)$ from .047 to .016.

A permanent consumption tax on the order of 5 to 10 percent is substantial. Thus our results provide quantitative support to the claims by Posner (2004), Parson (2007), Sunstein (2007), Allison (2004) and others that the risk of a national or global catastrophe is significant and should be taken more seriously by society. In particular, governments should devote greater resources to reducing that risk.

5 Conclusions.

We set out to find the mean arrival rate and impact distribution of possible catastrophic events that are national or global in scale. Rather than use historical data as others have done, we calculated these event characteristics as calibration outputs from a general equilibrium model. Thus we assumed that these characteristics are those perceived by firms and households, in that they are consistent with the data for key economic variables.

We found the annual probability of a catastrophe to be between 0 and about .04. A reasonable estimate would be in the middle of our range, i.e., around .02. This is close to Barro's (2006) estimate from historical data, but was obtained in a very different way. Our estimates of the impact distribution and expected loss should a catastrophe occur are tighter, but depend on the index of risk aversion (which we take to be between 2 and 4). However, the expected losses are large; about 26 to 32 percent if $\gamma = 2$.

We calculated as a "willingness to pay" measure the permanent tax on consumption that society would accept to reduce the annual probability of a catastrophe by some percentage. Using the mid-range estimate of .02 for the annual probability, a permanent consumption tax of about 9 percent would be justified if it could cut this probability in half. Even if the probability is lower than .02, our results suggest that governments should devote greater resources to reducing the risk and potential impact of a global catastrophe .

Our results also have implications for some of the behavioral parameters that are often used in macroeconomic and financial modeling. In all cases the calibrations yield values for the elasticity of intertemporal substitution that are well below 0.5. In addition, our results are consistent with the view that restricting preferences to expected CRRA utility is not a

bad approximation for modeling purposes.

Some caveats are clearly in order. Our model is simple and highly stylized. For example, we solved the social planner's problem for a representative firm with an AK production technology and adjustment costs, and a representative household with rational expectations. This is equivalent to a competitive equilibrium with a large number of identical firms and identical households, with the same production technology and preferences, so that we ignore heterogeneity among firms and households. And while our calibrations fit the basic economic aggregates, we do not formally test the model in a statistical sense. We also characterize catastrophic events in a simple way — a Poisson arrival with a constant mean arrival rate, and a permanent impact that follows a one-parameter distribution. These simplifications, however, make the model highly tractable, and provide an innovative approach to estimating the characteristics and policy implications of possible catastrophic events.

Appendix

A. Solution of Model.

Substituting the conjectured value function (10) into the consumption FOC (9) yields:

$$\rho C^{-\psi^{-1}} \frac{1}{(bK)^{(1-\gamma)(\omega-1)}} = \phi'(i)(bK)^{-\gamma} b .$$

Simplifying the above and using $c = C/K$, we have

$$c = \left(\frac{\rho}{\phi'(i)} \right)^\psi b^{1-\psi} . \quad (36)$$

Substituting this relationship between c and b back into the Bellman equation (8) yields the implicit equation (12), which gives the solution for the optimal i^* .

Let $\{M(t) : t \geq 0\}$ be the stochastic discount factor. From Duffie and Epstein (1992),

$$M(t) = \exp \left[\int_0^t f_V(C_s, V_s) ds \right] f_C(C_t, V_t) . \quad (37)$$

From the equilibrium allocation results,

$$f_C(C, V) = \phi'(i^*) b^{1-\gamma} K^{-\gamma} , \quad (38)$$

$$f_V(C, V) = -h , \quad (39)$$

where

$$h = -\frac{\rho(1-\gamma)}{1-\psi^{-1}} \left[\left(\frac{c^*}{b} \right)^{1-\psi^{-1}} \left(\frac{\psi^{-1}-\gamma}{1-\gamma} \right) - 1 \right] . \quad (40)$$

Using the equilibrium relation between b and c^* , we can simplify the above as follows:

$$h = \rho + (\psi^{-1} - \gamma) \left[\phi(i) - \frac{\gamma\sigma^2}{2} - \lambda\mathcal{E} \left(\frac{1 - Z^{1-\gamma}}{1-\gamma} \right) \right] . \quad (41)$$

Using Ito's lemma and the equilibrium allocation, we have

$$\frac{1}{M(t)} dM(t) = -h dt - \gamma [\phi(i^*) dt + \sigma dW(t)] + \frac{\gamma(\gamma+1)}{2} \sigma^2 dt + (Z^{-\gamma} - 1) dJ(t) . \quad (42)$$

The equilibrium restriction that the expected rate of change of $M(t)$ must equal $-r(t)$ implies the following formula for the equilibrium interest rate:

$$r = h + \gamma\phi(i^*) - \frac{\gamma(\gamma+1)\sigma^2}{2} - \lambda\mathcal{E} (Z^{-\gamma} - 1) . \quad (43)$$

Let $Q(K)$ denote the value of the capital stock and q be the corresponding Tobin's q . By homogeneity, $Q(K) = qK$. The equilibrium dividend is then $D(t) = C(t)$ for all t . The standard valuation methodology implies that $M(t)D(t)dt + d(M(t)Q(t))$ has an instantaneous drift of zero. Using Ito's lemma and simplifying yields an equation for q :

$$\frac{c^*}{q} = \rho - (1 - \psi^{-1})\phi(i^*) + \frac{\gamma(1 - \psi^{-1})\sigma^2}{2} + \frac{\lambda}{1 - \gamma}\mathcal{E} \left[(\psi^{-1} - 1) (Z^{1-\gamma} - 1) \right]. \quad (44)$$

Comparing with (12), we see that Tobin's q is given by $q = 1/\phi'(i)$. The expected rate of return on the aggregate equity claim is then

$$r^e = \rho + \psi^{-1}\phi(i^*) - \frac{\gamma(\psi^{-1} - 1)\sigma^2}{2} + \lambda\mathcal{E}(Z - 1) + \frac{\lambda}{1 - \gamma}\mathcal{E} \left[(\psi^{-1} - 1) (Z^{1-\gamma} - 1) \right]. \quad (45)$$

Therefore the aggregate risk premium rp is given by

$$rp = r^e - r = \gamma\sigma^2 + \lambda\mathcal{E} \left[(Z - 1) (1 - Z^{-\gamma}) \right]. \quad (46)$$

B. Consumption Tax.

Let c denote after-tax consumption-capital ratio $c = C/K$, so $c = (1 - \tau)(A - i)$. Substituting the conjectured value function $V(K; \lambda, \tau)$ given by (30) into the FOC (29) yields:

$$c^{1/\psi} = \frac{(1 - \tau)\rho}{\phi'(i)} b^{1/\psi-1}. \quad (47)$$

Substituting (47) for i into the Bellman eqn. (8) and simplifying, we can write the equilibrium consumption-capital ratio c^* as:

$$c^* = \frac{1 - \tau}{\phi'(i^*)} \left[\rho + (\psi^{-1} - 1) \left(\phi(i^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1 - \gamma}\mathcal{E} (1 - Z^{1-\gamma}) \right) \right]. \quad (48)$$

We also have the accounting identity: $c^* = (1 - \tau)(A - i^*)$. The optimal investment-capital ratio i^* thus solves:

$$A - i^* = \frac{1}{\phi'(i^*)} \left[\rho + (\psi^{-1} - 1) \left(\phi(i^*) - \frac{\gamma\sigma^2}{2} + \frac{\lambda}{1 - \gamma}\mathcal{E} (1 - Z^{1-\gamma}) \right) \right]. \quad (49)$$

Rewriting (47) and using i^* from solving (49), we have the following equation for b :

$$b(\lambda; \tau) = (c^*)^{1/(1-\psi)} \left[\frac{(1 - \tau)\rho}{\phi'(i^*)} \right]^{-\psi/(1-\psi)}. \quad (50)$$

Therefore,

$$b(\lambda; \tau) = (1 - \tau)^{1/(1-\psi)} (A - i^*)^{1/(1-\psi)} (1 - \tau)^{-\psi/(1-\psi)} \left[\frac{\rho}{\phi'(i^*)} \right]^{-\psi/(1-\psi)} = (1 - \tau)b^*(\lambda),$$

where the last equality follows from (11).

References

- Allison, Graham, *Nuclear Terrorism: The Ultimate Preventable Catastrophe*, Henry Holt & Company, 2004.
- Attanasio, Orazio P., and Annette Vissing-Jorgensen, “Stock Market Participation, Intertemporal Substitution and Risk Aversion,” *American Economic Review*, Papers and Proceedings, May 2003.
- Bansal, Ravi, and Amir Yaron, “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *Journal of Finance*, 2004. **59**, 1481–1509.
- Barro, Robert J., “Rare Disasters and Asset Markets in the Twentieth Century,” *Quarterly Journal of Economics*, 2006, **121**, 823–66.
- Barro, Robert J., “Rare Disasters, Asset Prices, and Welfare Costs,” *American Economic Review*, March 2009.
- Barro, Robert J., Emi Nakamura, Jon Steinsson, and José Ursúa, “Crises and Recoveries in an Empirical Model of Consumption Disasters,” unpublished working paper, 2009.
- Barro, Robert J., and José Ursúa, “Macroeconomic Contractions since 1870,” *Brookings Papers on Economic Activity*, Spring 2008. 255–350.
- Breeden, Douglas T., “An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities,” *Journal of Financial Economics*, 1979, **7**, 265–296.
- Duffie, Darrell and Larry G. Epstein, “Stochastic Differential Utility,” *Econometrica*, 1992, **60**, 353–394.
- Gabaix, Xavier, “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance,” NBER Working Paper No. 13724, January 2008.
- Gourio, François, “Disasters and Recoveries,” *American Economic Review*, May 2008, **98**, 68–73.
- Guvenen, Fatih, “Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective,” *Journal of Monetary Economics*, October 2006, **53**, 1451–72.
- Hall, Robert E., “Intertemporal Substitution in Consumption,” *Journal of Political Economy*, April 1988, **96**, 339–357.
- Hall, Robert E., “Reconciling Cyclical Movements in the Marginal Value of Time and the Marginal Product of Labor,” *Journal of Political Economy*, April 2009, **117**, 281–323.
- Lucas, Robert E, Jr, “Asset Prices in an Exchange Economy,” *Econometrica*, 1978, **46**, 1429–45.

- Parson, Edward A., “The Big One: A Review of Richard Posner’s *Catastrophe: Risk and Response*,” *Journal of Economic Literature*, March 2007, **45**, 147–164.
- Posner, Richard A., *Catastrophe: Risk and Response*, Oxford University Press, 2004.
- Russett, Bruce, and Miles Lackey, “In the Shadow of the Cloud: If There’s No Tomorrow, Why Save Today?” *Political Science Quarterly*, Summer 1987, **102**, 259–272.
- Russett, Bruce, and Joel Slemrod, “Diminished Expectations of Nuclear War and Increased Personal Savings: Evidence from Individual Survey Data,” *American Economic Review*, Sept. 1993, **83**, 1022–1033.
- Slemrod, Joel, “Fear of Nuclear War and Intercountry Differences in the Rate of Saving,” *Economic Inquiry*, Oct. 1990, **28**, 647–657.
- Sunstein, Cass R. *Worst-Case Scenarios*, Harvard University Press, 2007.
- Wachter, Jessica, “Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?” NBER Working Paper No. 14386, October 2008.
- Weitzman, Martin L., “Subjective Expectations and Asset-Return Puzzles,” *American Economic Review*, Sept. 2007, **97**, 1102–1130.