CEEPR **Center for Energy and Environmental Policy Research**

Electricity Transmission Pricing: How much does it cost to get it wrong?

by
Richard Green

04-020 WP **September 2004**

A Joint Center of the Department of Economics, Laboratory for Energy and the Environment, and Sloan School of Management

Electricity Transmission Pricing:
How much does it cost to get it wrong?

Richard Green [*]

University of Hull Business School,

Hull HU6 7RX

Tel +44 1482 465720

Fax +44 1482 466216

Email r.j.green@hull.ac.uk

September 2004

Economists know how to calculate optimal prices for electricity transmission. These are rarely applied in practice. This paper develops a thirteen node model of the transmission system in England and Wales, incorporating losses and transmission constraints. It is solved with optimal prices, and with uniform prices for demand and for generation, re-dispatching when needed to take account of transmission constraints. Moving from uniform prices to optimal nodal prices could raise welfare by 1.5% of the generators' revenues, and would be less vulnerable to market power. It would also send better investment signals, but create politically sensitive regional gains and losses.

Keywords: Electricity Transmission Pricing, Welfare Losses, Market Power
JEL: L94

---

[*] Support from the British Economic and Social Research Council under the project *Developing Competition in the British Energy Market*, R000236828, and the Leverhulme Trust, through the award of a Philip Leverhulme Prize, is gratefully acknowledged. I would like to thank the University of California Energy Institute and the Center for Energy and Environmental Policy Research at MIT for their hospitality, Nicholas Cordella for dedicated research assistance, and Paul Joskow, Bart McGuire, David Newbery and audiences at UCEI, INFORMS, the Stanford Energy Modelling Forum, and the University of Helsinki Systems Analysis Laboratory workshop for helpful comments.

Electricity Transmission Pricing:
How much does it cost to get it wrong ?

Richard Green

University of Hull

I. Introduction

For more than twenty years, economists have known how to calculate the optimal locational prices for electricity. That knowledge seems to have been ignored by the designers of many electricity markets. Chile, New Zealand and some US markets have adopted the optimal system of nodal spot pricing, but all the electricity markets in Europe have adopted much simpler systems, and several have no locational pricing at all. This paper asks how far welfare is reduced by the choice of such simpler pricing rules. In a simplified model of England and Wales, representing the industry in 1996/7, it compares prices, profits and consumer welfare under a system of nodal prices, uniform prices, and a hybrid in which generators face nodal prices but consumers face a uniform price.

The first-best price of electricity at each point on a network (node) equals the marginal cost of providing electricity at that node. The electricity must not only be generated, but it must also be delivered to that node, taking account of transmission constraints and electrical losses. If transmission constraints are binding, so that the amount of power flowing through a line is at the limit which safety allows, then cheap but distant generation may have to be replaced with more expensive local generation, in order to reduce power flows. In the constrained area, the optimal price of electricity rises to the marginal cost of the local generation, or to the level needed to ration demand to the amount of electricity available. Even if there are no constraints, some power will be lost in the transmission system (dissipated as heat), and prices should reflect the fact that it is more expensive to provide electricity at the far end of a heavily loaded line than close to a power station. Transmission Congestion Contracts (Hogan, 1992) could be used to hedge spatial price differentials, and to help co-ordinate investment.

These principles are well-known, but few electricity systems have adopted them. Chile, New Zealand and a small number of US power pools have markets which are based upon nodal spot prices, which is also proposed in the Federal Energy Regulation Commission’s Standard Market Design, but almost every other country in the world uses a simplified system of transmission pricing. Nodes may be grouped together into zones, and the price differentials between zones are calculated from simplified models. Other systems still see transmission as an “overhead” cost, and use simple “wheeling rates” to calculate payments if one company imports power from a second over the lines of a third. These payments are typically based upon the volume of the flow and the length of its contracted route (the MW-mile approach), and frequently ignore the fact that any transaction in an interconnected system will affect power flows on all the other networks in that system. Green (1997) discusses the pricing rules then adopted in eight electricity systems, assessing them against economic and political criteria. One common theme was that these rules tended to produce lower price differentials than would be associated with optimal spot prices.

How important are the differences between the relatively simple rules adopted in practice, and the prices which an optimal system would produce? One of the main economic functions of a price system is to signal the opportunity cost of alternative courses of action. On the demand side, an agent should buy something if it is valued at more than its price, while a supplier should produce it if this can be done for less than its price. If buyers and suppliers face the same prices, their independent decisions will ensure that the value of output at the margin is just equal to its marginal cost, which is optimal. If prices are above marginal costs, then too little of a good will be consumed and produced, while too much will be produced if prices are below marginal costs.[1] The wrong prices can also lead to inefficient "bypass" as agents have an incentive to leave the market, and arrange deals at prices closer to their costs.[2]

This paper takes a simplified model of the electrical system in England and Wales, calculates optimal prices and quantities, and compares the outcomes with those that simpler rules would produce. The model has thirteen nodes, with demand at every node and generation at most of them. It is solved for different levels of demand, and of generator availability, and a weighted sum of the results is used to give the impact over a year as a whole. In particular, we measure average prices, generators' profits, and the change in consumer surplus relative to the first-best case. We also look at way in which the geographical pattern of prices responds to the pricing rules.

Green (1994) used a similar model to examine changes in the total cost of generation once transmission losses were reflected in the dispatch, but ignored transmission constraints. Bialek *et al.* (2003) report on a similar study undertaken for the UK Department of Trade and Industry on the impact of introducing zonal losses charging across Great Britain. Macatangay (1997) takes account of constraints (but not losses) and calculates the price for each zone on the NGC system as the dual value in a power flow optimisation. Ilic *et al* (1997) use a detailed model of New England to show how the payments for seven hypothetical wheeling transactions would change between three different cost allocation rules. None of these papers studies the impact of spatial pricing on demand. This paper incorporates demand responses in a model which takes account of both losses and transmission constraints. An earlier version carried out a similar study (Green, 1998), but reported the results only for representative hours.

---

1 This is a slight simplification. When there are several goods, slightly too much of one good may be produced, even though its price is *above* marginal cost, because other prices are further away from costs.

2 We will return to this issue in the conclusions.

This paper also studies the interaction of market power and transmission pricing. In electricity networks, the exercise of market power can take counter-intuitive forms: it may involve *increasing* output at some locations, in order to tighten a transmission constraint and raise prices elsewhere (Cardell *et. al.* (1997)). Borenstein *et al* (2000) show how small increases in transmission capacity can lead to significant reductions in market power, once congesting a link becomes unattractive. Oren (1997) suggests that actively traded contracts could reduce the incentives for this kind of behaviour. Joskow and Tirole (2000) show how generators' holdings of transmission contracts affect their behaviour, and that these can worsen market power. For example, a generator in an importing region could hold transmission contracts that increase its exposure to the local price, enhancing its incentive to raise that price. Gilbert, Neuhoff and Newbery (2004) show that the impact of transmission contracts depends on the design of the markets in which they are traded – generators will not obtain contracts that worsen their market power in a uniform price auction, but may do so in a discriminatory auction, for example. Ehrenmann and Neuhoff (2003) simulate the market in Belgium and the Netherlands and obtain lower prices when the markets are integrated than when rights to use the cross-border interconnectors are traded before the energy markets open.

This paper does not examine the impact of contracts, but does allow the largest generator to act strategically. This generator sometimes reduces output to create an import constraint into an area where it owns most of the local generation, allowing it to raise prices well above their level in the rest of the country. Under one pricing rule, it also tries to raise its output in a second area, which is subject to an export constraint, in order to increase the compensation that it obtains for reducing output back to the feasible level.

The next section of the paper gives a brief outline of the theory of transmission pricing. Section III describes the model. Section IV describes the three pricing rules that are compared. Section V shows how welfare changes with these rules. Welfare is highest when the model is solved with optimal prices (one for each node), and lowest with a single price for demand and one for generation. The single-price system is based upon that used in the Pool in England and Wales, so that generators were made to change their outputs in order to ensure a feasible dispatch, but were compensated for the opportunity costs of doing so. Since political considerations sometimes dictate that consumers should face a uniform price, even if locational pricing would be acceptable for generators, the model is also solved with varying prices for generators, but a uniform price (in each time period) for demand. Section VI considers the impact of market power. Section VII concludes.

II. Optimal Transmission Pricing

The theory of spot pricing is set in detail out by Schweppe *et al* (1988). The interested reader should refer to that book, or the shorter exposition given by Hsu (1997) for details, for this section will merely state their key formula for the spot price at any point on a network, and concentrate on giving the underlying intuition for it.

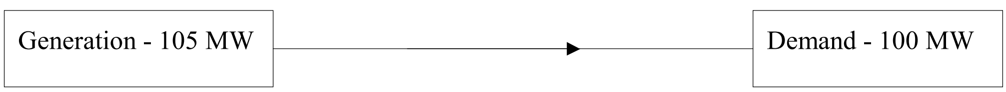
The optimal prices for electricity transmission can be seen as arising from the problem of maximising the net welfare obtained from electricity consumption, subject to a number of constraints. This net welfare is equal to the benefit from consuming electricity, less the cost of generating it. For simplicity, we will ignore any variable costs which are not manifested in a need for increased generation.[3] The constraints to be met are that total generation must equal total demand, plus losses, and that the flow along each transmission line must be less than the capacity of that line. The flows depend on the levels of generation and demand at each node. For practical applications, they are derived from a load flow model such as the DC load flow model, which is outlined below. We can write a simplified version of the problem as a Lagrangean:

$$
\begin{aligned}
\underset{\underline{d},\underline{g}}{\text{maximise}} \quad & \sum_k B(d_k) - \sum_j C(g_j) \\
& - \mu_{\mathrm{e}} \left(\sum_k d_k + l - \sum_j g_j \right) \qquad \text{(energy balance constraint)} \\
& - \mu_i^{\mathrm{QS}} \left(|z_i| - z_i^{\max} \right) \qquad \text{(line flow constraints)}
\end{aligned}
\qquad (1)
$$

where d_k represents the demand at node *k*, g_j represents the generation at node *j*, and z_i represents the flow along line *i*. Losses in transmission are equal to *l*. The maximum flow allowed on line *i* is given by $z_i^{\max}$. The Lagrangean multiplier on the energy balance constraint is μ_e, while the multiplier on the flow constraint for line *i* is μ_i^{QS}. The first-order conditions to this problem can be manipulated to give the price at node *k*:

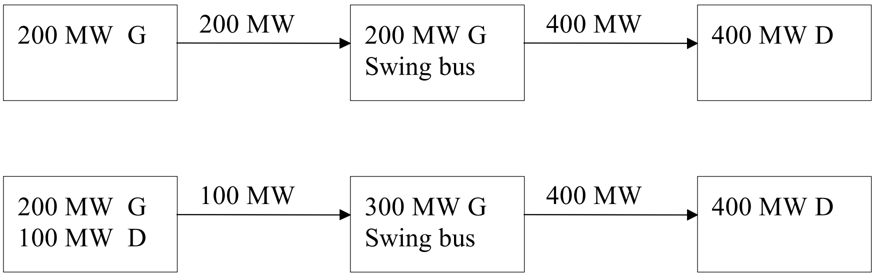
$$
p_k = \mu_e \left[1 + \frac{\partial l}{\partial d_k} \right] + \sum_i \mu_i^{QS} \frac{\partial z_i}{\partial d_k} \qquad (2)
$$

The multiplier on the energy balance constraint can be interpreted as the marginal cost of generation at the "swing bus", the location of the marginal generator - this would be the cost of providing another unit of electricity in a system unaffected by losses or constraints. In practice, however, some electricity is lost in transmission, and so more or less than 1 MW will have to be generated in order to deliver 1 MW. If demand at node *k* leads to an increase in flows, losses will increase. Taking the simplest possible example, of a two-node system, we might have:



[3] In other words, we assume that the cost of transmission maintenance, for example, does not depend on the level of power flows.

The average loss on this line is 5% of the power delivered, but the loss is proportional to the flow squared, and so the marginal loss will be 10%, twice the average. If the marginal cost of generation (which should be the price at the generation node) is £20/MWh, then the price at the demand node should be £22/MWh. It is possible for an increase in demand to reduce system losses, however, if it reduces the flows along some lines. In the example below, 100 MW of demand at the left-hand node is met by an extra 100 MW of generation at the central node, and the flow along the left-hand line decreases. The price at the left-hand node should be less than at the central node, reflecting this.



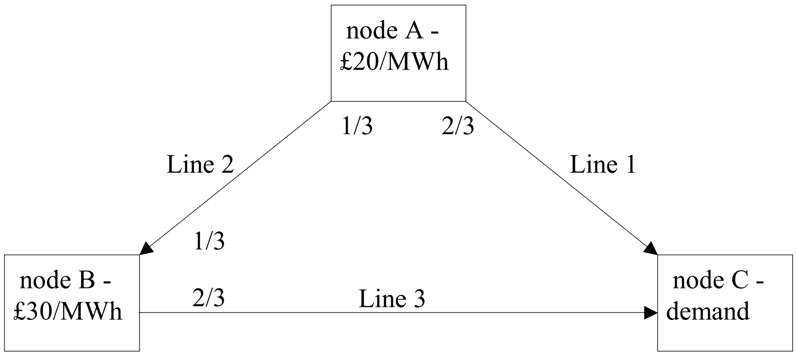
The final term in equation (2) concerns the cost of constraints. Returning to the two-node network, if the line between the nodes was operating at its maximum capacity, then any further demand at the right-hand node would have to be met by generation at that node. If the marginal cost of generation at that node was £30/MWh, this should be the price of electricity at that node. A 1 MW increase in the capacity of the link would allow another MW of power to be delivered from the left-hand node at a cost of £22/MWh, so that the shadow price of the constraint is £8/MWh. Each 1 MW increase in demand causes a 1 MW increase in the flow across the constrained link, and so the final term in equation (2) would equal 1 in this example. In this simple example, the price at each node is equal to the marginal cost of the generation at that node, and the price equation is basically a way of decomposing the difference between them into the cost of losses and the cost of the constraint.

To solve more complicated examples, we must be able to derive the power flows in a "meshed" network, in which there is more than one route between some pairs of points. The DC Load Flow model used in this paper is an approximation, but a reasonable one for our purposes. The model is discussed in the appendix, but the key equation is

$$\underline{z} = R^{-1}A\left(A^{T}R^{-1}A\right)^{-1}\underline{y} \qquad (3)$$

where $\underline{z}$ is the vector of power flows along the L lines and $\underline{y}$ is the vector of net injections at the N-1 nodes excluding the swing bus. R is an $L\times L$ diagonal matrix of the resistances on each line, and A an $L\times N$-1 matrix showing the connections between lines and nodes, known as the network incidence matrix. The key results from the model (or any other realistic model of line flows) are that power flows between two nodes will be shared among all the lines connecting those nodes, directly or indirectly, but that the distribution of the flow between any two routes will be inversely proportional to the relative resistance of those routes.

In the example below, in which all three lines have the same resistance, two-thirds of the power generated at node A will flow through line 1, and one-third through lines 2 and 3. Two-thirds of the power generated at node B will flow through line 3, and one-third through lines 1 and 2. Since flows on line 2 are defined to be from A to B, (as shown by the arrow) the output from B which is flowing towards A will actually be measured as a negative flow (or, equivalently, a reduction in the flow from A to B).



Assume that there are no losses, to keep the example straightforward. If there were no constraints, and sufficient capacity at node A, the price of electricity would be £20/MWh at all three nodes. If there is a binding transmission constraint on line 1, however, then a 1 MW increase in demand at node C can only be met by *reducing* generation at A by 1 MW, and increasing generation at B by 2 MW. The additional output from B would cause ⅔MW to flow through line 1 (⅓ of the extra output), but the reduction in output from A would decrease the flow along the congested line by ⅔MW. The marginal cost of meeting the demand is £40/MWh - (2 MWh from B at £30 each, but saving 1 MWh from A which would have cost £20).[4] This example shows that the cost of delivering 1 MW to some points on the network could be greater than the marginal cost of generation at any individual node - other examples could be constructed in which the delivered cost of power is negative. Since dealing properly with constraints can produce such strong price signals, studying the cost of ignoring them seems to be a worthwhile project.

[4] This price (£40/MWh) could also be decomposed as in equation (2). The marginal cost at node A is £20/MWh, which is equal to μ_e. Adding 1 MW of capacity to line 1 would allow us to replace 3 MWh of generation at B (costing £90) with 3 MWh of generation at A (costing £60), a saving of £30, giving us the value of μ_i^{QS}. (The reduction at B reduces the flow on line 1 by 1 MW, which allows half of the increase caused by the extra generation at A - the other half comes from the new capacity). Increasing demand at C by 1 MW will increase the flow on line 1 by ⅔MW. We get £40 = £20 + £30 × ⅔.

III. The Model

This paper studies the impact of different transmission pricing schemes on a simple model, but one which is intended to represent the main flows over the national grid system in England and Wales. That system has 14,000 km of 275 kV and 400 kV transmission lines, in 400 circuits which connect 200 substations. The (winter) peak demand on the system is presently around 50 GW. The main power flows are from power stations in the north of England and the Midlands to the south, although there are several large power stations in the south-east, near the Thames Estuary.

The National Grid Company (NGC) has divided its system into a number of zones for charging generators: the boundaries of the zones generally coincide with groups of circuits which are heavily loaded and might be constrained under some operating conditions. Our model is based upon the zones used in the mid-1990s, and uses one node to represent each zone. The exceptions are that the northernmost zone is split into two, and that two southern zones with no generation are combined with their neighbours. Twenty-one lines link our thirteen nodes, so that all zones which were directly connected are linked.

The model is based upon projections for the summer and winter of 1996/97, made by NGC in April 1996. The company publishes a seven-year projection of demand and capacity each year, together with data on the transmission system. Table 1 shows the distribution of

Table 1: Zones, Generation and Demand (GW)

Zone	Name	Generation	Peak Demand		Marginal
		(Winter)	(Winter)	(Summer)	Loss
0	North [-Western]	0.6	0.3	0.2	
1	North [-Eastern]	4.3	2.4	1.7	+ 5%
2	Yorkshire	9.4	5.6	3.9	+ 3%
3	N Wales and W Lancs	7.8	4.2	3.0	+ 3%
4	E Lancashire	0.0	2.8	2.0	+ 1%
5	Nottinghamshire	4.1	0.5	0.4	+ 1%
6	West Midlands	4.5	7.0	4.9	- 1%
7	East Anglia	3.0	4.8	3.3	- 3%
8	West and Wales	3.8	4.5	3.2	- 6%
9	[Thames] Estuary	8.5	2.5	1.8	- 5%
10	London [Inner and Outer]	2.0	8.7	6.1	- 5%
12	South Coast	0.5	3.8	2.6	- 9%
13	Wessex and Peninsula	1.6	2.8	1.9	- 9%

Source: NGC, (1996)

Table 2: Generation Capacity (GW)

Zone	Name	Must-run	Variable	Total
0	North [Western]	0.6	0.0	0.6
1	North [Eastern]	3.9	1.1	5.0
2	Yorkshire	3.5	8.0	11.4
3	N Wales and W Lancs	5.9	4.1	10.0
4	E Lancashire	0.0	0.0	0.0
5	Nottinghamshire	0.0	4.9	4.9
6	West Midlands	0.2	5.2	5.4
7	East Anglia	3.6	0.0	3.6
8	West and Wales	1.1	3.5	4.6
9	[Thames] Estuary	4.5	5.4	9.9
10	London [Inner and Outer]	1.7	0.9	2.6
12	South Coast	0.0	0.7	0.7
13	Wessex and Peninsula	1.7	0.2	1.9
		26.8	33.9	60.7

Source: NGC (1996). Figures do not sum to totals due to rounding

generation and demand at the winter peak in 1996/7, together with demand at the summer peak. Generation in zones 0 and 1 includes 0.6 GW of imports from Scotland in each zone, while generation in zone 9 includes 2 GW of imports from France. The table also gives NGC's zonal predictions of the marginal losses from extra generation in 2002/3 (the only year for which the figures were given): these were used as a guide when calibrating the model, although it was not possible to replicate them exactly.

Table 2 gives more information on the generation which is located at each node. Two types are identified: must-run and variable. Both nuclear and combined-cycle gas turbine stations are classed as must-run plants, submitting very low bids and running as often as they are available. This reflects the way in which the earlier gas turbine stations were bid into the Pool, partly determined by their contracts; later stations have been more flexible. The variable stations are assumed to have a linear marginal cost function: the most expensive capacity has a marginal cost of £60/MWh. Figure 1 shows the assumed marginal cost function for zone 9.

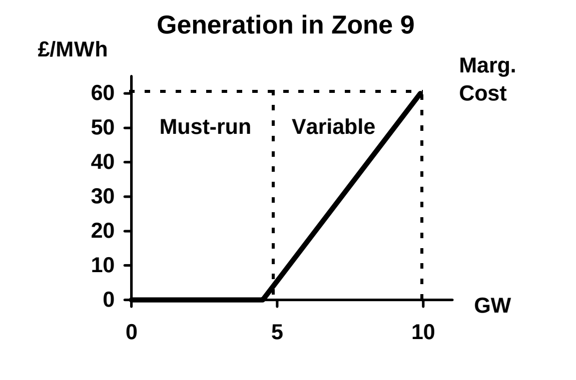
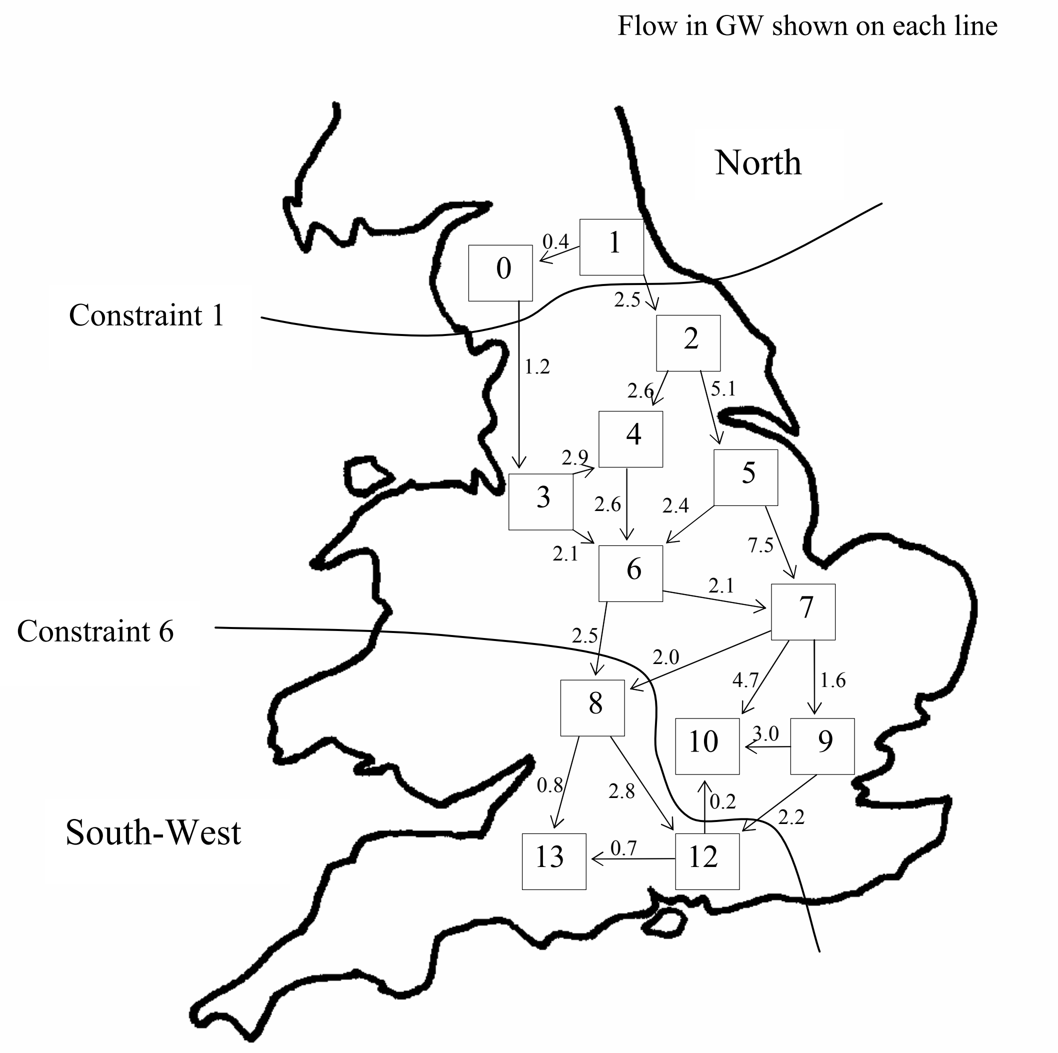


Figure 1

This form of marginal cost function is a simplification, because the marginal cost function for any one power station would be roughly horizontal, while a node with several stations would have a step function. This would imply that small changes in prices would frequently have no effect on the pattern of generation, but that a change would occasionally have a discontinuous impact as one station became cheaper than another.[5]

In the presence of discontinuities, the cost of a distortion may be very sensitive to the size of that distortion, since small distortions will be costless. Our formulation gives more opportunities for adjusting generation in response to prices. Generation availability was de-rated to 90% of the raw figure in winter, and 70% in summer, to reflect outages for maintenance (mostly forced in winter, with planned maintenance concentrated in the

Figure 2: Simplified Version of the Transmission System in England and Wales



[5] This could lead to further complications, if the new distribution of generation affected power flows and the associated prices. In practice, the price at each of several nodes might be set to equal the marginal cost of a station at that node, and the output from each station would then have to be adjusted until the resulting power flows meant that the selected prices were the correct spot prices, given the power flows.

Table 3: Lines and Flows

Line	From Zone	to Zone	Resistance (×1000)	Flow (GW)
1	1	0	8.775	0.4
2	1	2	9.000	2.5
3	0	3	9.000	1.2
4	3	4	2.025	2.9
5	2	4	3.450	2.6
6	2	5	2.250	5.1
7	3	6	8.325	2.1
8	4	6	4.500	2.6
9	5	6	3.900	2.4
10	5	7	2.250	7.5
11	6	7	3.675	2.1
12	6	8	9.075	2.5
13	7	8	7.500	2.0
14	7	10	8.625	4.7
15	7	9	16.125	1.6
16	9	10	4.875	3.0
17	9	12	5.925	2.2
18	12	10	8.250	0.2
19	8	12	8.250	2.8
20	8	13	32.250	0.8
21	12	13	3.375	0.7

summer).

Demand is price sensitive, with a constant elasticity of -0.25 at each node, representing a "medium-term", rather than a "short-term", response. This is the elasticity with respect to the price of generation: the implied elasticity with respect to the final price will be rather greater. There are ten sets of demand curves in each season, representing different load levels. Each set is obtained by scaling down the regional peak demands by a common factor, chosen to match points on the seasonal load-duration curve.[6] The demand curves are anchored so that these quantities are demanded at the (uniform national) prices which the Pool pricing system, as used in the 1990s, would have produced.

Figure 2 shows how the network of lines links the thirteen nodes in the model. With two exceptions, electricity flows from a lower to a higher numbered zone: the reverse flows are from zone 1 (the western half of NGC's zone 1) to zone 0, and from zone 12 to zone 10.

[6] For example, the winter peak demands are created by scaling the peak demand by 1.0, 0.9, 0.8, 0.76, .073, 0.71, 0.68, 0.64, 0.6 and 0.54. In practice, the figures in table 1 were first reduced by 1.8% to take account of transmission losses, which are not separately identified in this table of the NGC source document.

Table 4: Constraints on the NGC system

Boundary	Lines	Zones affected		Max rating (GW)
1	2,3	Exports from	0,1	2.1
2	6,7,8	Exports from	0 – 4	9.2
3	10,11,12	Exports from	0 – 6	11.0
4	17,-18,19,20	Imports to	12,13	7.7
5	3,5,6	Exports from	0,1,2	6.8
6	12,13,17,-18	Imports to	8,12,13	7.8
7	20,21	Imports to	13	4.0
8	14,16,18	Imports to	10	10.5
9	-15,16,17	Exports from	9	8.5
10	1,2	Exports from	1	2.0
11	7,8,9,-11,-12	Imports to	6	6.0
12	3,5,9,10	Exports from	0,1,2,5	11.5

Source: NGC (1996)

Table 3 describes the lines which link these zones: the resistances have been chosen so that the flows between zones, and the marginal losses incurred on demand in each zone, are close to those given in NGC (1996). These losses are only published for the final year of the projection, and so the network model was initially calibrated for 2002/3, using demand and generation figures for that year. One adjustment was then made to represent the system in 1996/7. NGC planned to double the capacity of the lines between zone 1 and zone 2 before 2002, and so the resistance between those zones was doubled to reflect the state of the system before this investment.

The transmission system in England and Wales suffers from a number of constraints. Some of these are local and cannot be modelled at the lower level of detail in this paper: a particular station is required to run to make the adjoining lower-voltage system more reliable. Other constraints relate to the flows across a number of heavily loaded boundaries: although NGC uses a different methodology to define its charging zones, their boundaries generally coincide with some of these critical system boundaries. Table 4 shows these constrained circuits and their maximum ratings.[7] While constraints 2 and 12 were occasionally binding, with small impacts on prices, the most important constraints for our purposes were numbers 1 and 6. Constraint 1 separates the North from the rest of the market, while constraint 6

[7] The boundary ratings are set so that the system can absorb the loss of any circuit without overloading the remaining lines. It is usual for a few lines to be unavailable, reducing the system's capacity to take further outages, and hence the safe boundary flows. Circuit ratings were accordingly reduced to raise constraint costs to the level seen in the mid-1990s.

isolates the South of Wales and the South-West of England. These two constraints are shown in Figure 2.

This small-scale model cannot be a fully accurate representation of the NGC system, but it will behave in similar ways in response to small changes in generation or demand. The aim of this paper is to consider whether different transmission pricing systems are likely to have an important practical impact on the electricity industry. This is a wider question than calculating prices in particular locations at particular times, although we need to do that before we can come to an answer. To answer the wider question, however, we do not need an exact representation of any one system, as long as our model incorporates the kind of responses which would be found in a real system.

IV. The Pricing Rules

The model was solved for a range of demand levels under "summer" and "winter" conditions (plant and transmission availability), and the results aggregated to give figures for the year as a whole. Three pricing rules were used. The first was optimal pricing: a single price at each node, for both generation and demand. The second pricing rule had separate prices at each node for generation, but a uniform national price for demand. The third pricing rule was that used by the Pool until 2001: one national price for generation, and one price for demand. In this case, "counter-trading" was used to manage constraints. Generation was reduced at nodes on the "export" side of a constraint, and increased it at the others, in proportion to the amount of "variable" capacity at each node, reducing the flows to acceptable levels. While normal generation is paid for at the zonal prices, constrained on generation is paid its cost (higher than its zonal price). Generation which is constrained off is also sold back to the generator at cost, (it must be sold back, since it had already been "paid for" under the rules used in England and Wales). If the generator had been bidding its marginal cost, it would neither gain nor lose from this transaction. If necessary, "must-run" generation could also be reduced to ensure that net exports from a node were within acceptable limits. (Similar reductions were sometimes needed with the second pricing rule, in cases when net exports were excessive, even with a local generation price of zero.) We will continue to use $g_j(p_j)$ to represent the generation that would be offered at node j at a price of p_j, and add c_j for changes in output due to constraints. The total output at node j is thus $g_j(p_j) + c_j$.

Note that all demand in an area is assumed to pay the same price, and so there will be no demand response unless that price changes. In practice, NGC has contracts with some consumers who are willing to reduce their demand on request, in return for a suitable payment. The primary purpose of most of these contracts is to provide reserve, in order to keep generation and demand in balance after a plant failure. Some contracts, however, are with customers who are well-placed to ease a (short-lived) constraint by reducing their demand, or even by increasing it. To the extent that NGC is able to contract directly with some of the more price-sensitive consumers, the quantities actually produced and consumed may be close to the efficient levels even if the price signal sent to the remaining (less price-sensitive) consumers is incorrect. The assumption of uniform pricing within an area may therefore overstate the welfare losses from inefficient pricing.[8]

The pricing rules can be related to the welfare problem stated in section II. The first rule, optimal pricing, is simply the solution to equation (1). The second rule is the solution to the closely related problem below. The additional constraint relates to the "merchandising surplus" which the transmission company makes with the optimal pricing rule. On average, demand exceeds generation at the higher-priced nodes, and the resulting surplus (*R*) can be put towards the fixed costs of the transmission network. The remaining costs are recovered through (distorting) tariffs. In order to compare welfare on a like with like basis, we require the transmission company to set the (uniform) demand price so that it makes the same surplus.

$$\begin{aligned}
\underset{p,\underline{p_j}}{\text{maximise}} \quad & \sum_k B(d_k(p)) - \sum_j C(g_j(p_j)+c_j) \\
& - \mu_e \left(\sum_k d_k + \text{losses} - \sum_j g_j + c_j \right) && \text{(energy balance constraint)} \qquad (4) \\
& - \mu_i^{QS} \left(|z_i| - z_i^{\max} \right) && \text{(line flow constraints)} \\
& - \rho \left(\sum_k p d_k(p) - \sum_j p_j g_j(p_j) - R \right) && \text{(total transmission surplus equal to } R\text{)} \\
& - p_j c_j && \text{(generation only constrained off at zero price)}
\end{aligned}$$

Applying the third pricing rule cannot be based on an optimisation that maximises welfare, since the solver would use counter-trading to equalise the marginal cost of generation in each zone, taking us back towards the results of the second rule. Instead, the solver was instructed to balance generation and demand, earning the same transmission surplus as with optimal pricing, and counter-trading only where a constraint would otherwise be breached. Generation on the two sides of a constraint was increased or decreased in proportion to the amount of variable capacity. In other words, if output had to be reduced by 0.4 GW to the north of a constraint, and half of the variable generation to the north of that constraint was in a given zone, then output in that zone would be reduced by 0.2 GW. In the absence of losses, this would minimise the cost of the adjustments, which is NGC's objective. The payment for the adjustment is equal to the production cost of the adjusted generation, less the production cost of the unadjusted generation. The constraints to be met were thus:

$$\sum_k d_k(p_d) + \text{losses} - \sum_j g_j(p_g) + c_j = 0 \quad \text{(energy balance constraint)}$$

[8] I am indebted to Paul Joskow for this point.

$$|z_i| \le z_i^{\max}$$ (line flow constraints)

$$\sum_k p_d d_k(p_d) - \sum_j p_g g_j(p_g) - \sum_j C(g_j(p_g) + c_j) - C(g_j(p_g)) = R$$ (total transmission surplus equal to *R*)

V. Results

Table 5 gives the results for our central case, with marginal cost bidding and a demand elasticity of 0.25. The average revenue is lowest, and the total output highest, with optimal pricing, while the Pool system of uniform pricing has the highest average revenue and lowest level of output. Despite the lower level of output, generation costs are higher with the two sub-optimal pricing systems. Under the Pool system, it is possible to identify the cost of constraints, as represented by NGC's counter-trading payments. The other pricing systems do not allow us to calculate a similar figure, although the extent of constraints can be seen in the coefficient of variation in generation prices. The first row is based on calculating the coefficient of variation across the zones for each demand level, and then taking the average of these coefficients. By construction, there is no variation under the Pool system, whereas the other two systems involve significant regional variations in prices. Much of this variation comes from a binding export constraint covering the two northernmost zones, and the rest from transmission losses. The variations (in the generators' prices) are greater when consumers face a uniform price, as the only way to reduce net exports from the north is to reduce generation. In many hours, the price has to drop to zero to achieve this. When the consumers' price varies as well, the reduction increases demand, reducing net exports and allowing equilibrium to be reached at a higher price. The second set of coefficients of variation shows that there is significant variation over time as well.

Welfare, defined as the unweighted sum of consumer surplus and profits, is £65 million per year lower with a uniform price for demand than with optimal pricing. This is equal to 0.8% of the revenue from optimal pricing. With uniform prices for both generation and demand, welfare is lower by £119 million, or 1.5% of the revenue from optimal pricing. Generators are worse off when consumers pay a uniform price, in part because the northern export constraint means that some generation is bought at a very low price – the demand response under optimal pricing allows a higher price in this area. Northern consumers accordingly lose from the move to a uniform price, but the gains to consumers elsewhere must exceed these losses, since consumer surplus rises overall. When both consumers and generators face uniform prices, however, generators are better off than with optimal pricing, and consumers worse off. Generators in export-constrained areas receive "lost profit" payments for the output that they are not able to sell, and this helps to raise their revenues.

Table 6 and Table 7 present results using two other values of the demand elasticity, as a sensitivity analysis. To the extent that price differentials arise from congestion which is

Table 5: Base case results

Pricing System	Optimal	Zonal (for Generators)	Uniform
Demand (TWh)	289.8	287.0	285.5
Losses (TWh)	1.53	1.59	1.60
Total Revenue (£m)	7936	7909	8043
Average Revenue (£/MWh)	27.38	27.56	28.17
Cost of counter-trading (£m)			142
Coefficient of Variation of prices:			
Within hours	0.28	0.42	0.00
Across and within hours	0.56	0.66	0.50
Changes relative to optimal pricing:			
Welfare (£m)		-65	-119
(% of revenue)		-0.8%	-1.5%
Consumer surplus (£m)		36	-205
(% of revenue)		0.5%	-2.6%
Generator profit (£m)		-102	86
(% of revenue)		-1.3%	1.1%
Cost of generation (£m)		74	20
(% of revenue)		0.9%	0.2%

only in place for part of the time, a lower elasticity might be appropriate, although we would expect a greater response to "permanent" differentials reflecting marginal losses. With a lower elasticity of –0.1, the optimal prices vary more, as demand is less responsive to changes in price. (When the price to consumers is uniform, however, the spatial variation of generation prices is very insensitive to the demand elasticity.) Although the efficient level of price variation is greater, the welfare impact of moving away from this system is smaller. Moving to a uniform price for demand reduces welfare by 0.4% of revenue; having a uniform price for generation as well reduces welfare by 1.1% of revenue. Having a uniform price for demand alone is good for consumers and bad for generators, as before, although the changes are smaller than in the base case. The Pool system is bad for consumers and good for generators, and in this case the impacts are greater than in the base case. Analogous results hold with a higher demand elasticity of –0.4. The optimal variation in prices is lower, but the cost of a non-optimal pricing system is greater.

VII. Market Power

It is well-known that electricity markets are vulnerable to market power. Does the choice of transmission pricing system affect this vulnerability? A full-blown simulation of an

Table 6: Low elasticity results

Pricing System	Optimal	Zonal (for Generators)	Uniform
Demand (TWh)	289.5	286.0	285.2
Losses (TWh)	1.55	1.58	1.60
Total Revenue (£m)	7879	7869	8093
Average Revenue (£/MWh)	27.22	27.51	28.37
Cost of counter-trading (£m)			142
Coefficient of Variation of prices:			
Within hours	0.37	0.42	0.00
Across and within hours	0.60	0.65	0.50
Changes relative to optimal pricing:			
Welfare (£m)		-35	-88
(% of revenue)		-0.4%	-1.1%
Consumer surplus (£m)		12	-291
(% of revenue)		0.2%	-3.7%
Generator profit (£m)		-47	203
(% of revenue)		-0.6%	2.6%
Cost of generation (£m)		36	11
(% of revenue)		0.5%	0.1%

oligopolistic market is beyond the scope of this paper, but this section uses a somewhat simplified approach to illustrate the impact of abusive behaviour by the largest company in the market. In 1996/7, National Power owned nearly one-third of the industry's capacity, as shown in table 8. While the rest of the industry continues to bid at marginal cost, we will allow National Power to vary the slope of the bid functions that it submits for its 16 GW of "variable" capacity. The market operator will follow the same pricing rules as before, treating National Power's submitted bid functions as true measures of its costs – the correct (original) cost functions will of course be used when calculating the actual level of welfare.

With the optimal pricing rule, an iterative process was used to obtain the generator's profit-maximising strategy. The transmission model was run to produce prices and quantities, and note which boundaries were constrained, for a given set of bid slopes. The generator then chose its profit-maximising bid slopes for a simplified version of the price-setting process, based on a linearisation around the previous outcomes. The full model was then used to calculate an accurate set of prices (since losses are non-linear in flows), and the process continued until it converged, which generally happened quickly. While this process only finds a local optimum for the generator, the process was also started with the company creating congestion on particular boundaries by the choice of its bid slopes, and the profits from these local optima were compared with those from the unconstrained market. The strategy finally used was the one that gave National Power its highest profits. This involved

Table 7: High elasticity results

Pricing System	Optimal	Zonal (for Generators)	Uniform
Demand (TWh)	290.0	287.8	285.8
Losses (TWh)	1.51	1.60	1.60
Total Revenue (£m)	7988	7958	8030
Average Revenue (£/MWh)	27.55	27.64	28.10
Cost of counter-trading (£m)			142
Coefficient of Variation of prices:			
Within hours	0.22	0.42	0.00
Across and within hours	0.53	0.66	0.50
Changes relative to optimal pricing:			
Welfare (£m)		-82	-135
(% of revenue)		-1.0%	-1.7%
Consumer surplus (£m)		51	-149
(% of revenue)		0.6%	-1.9%
Generator profit (£m)		-134	14
(% of revenue)		-1.7%	0.2%
Cost of generation (£m)		103	28
(% of revenue)		1.3%	0.4%

submitting high bids for its plants in zones 8 and 12 when demand was high, in order to congest the boundary around the south-west of England and Wales (boundary 6). When demand was lower, the company would have had to give up too much output to make it profitable to create the constraint, and allowing a (nearly) national market was better.[9]

Within each group of zones, the profit-maximising price-cost mark-ups were almost identical. Identical mark-ups were imposed when finding the strategies used with the other two pricing rules, which was done through a grid search, running the full model to calculate profits for a range of slopes.[10] The most profitable strategy was then applied, subject to two constraints. The generator could not submit a price more than ten times its true cost in any zone, and it had to keep the highest prices below £120/MWh. These constraints were applied to reflect the presence of a regulator in the market, and prevent the company from earning (nearly) unbounded profits when its output was required to meet a constraint. Table 9 shows the results.

[9] The northernmost zones were always separated by a binding constraint, and National Power did not own enough of the plant in the region to justify a strategy of bidding low to relax the constraint.

[10] With these pricing rules, the generator was sometimes close to a capacity constraint in some zones. These constraints interacted with the linearised model in a way that meant profit-maximising strategies in the model were far from profit-maximising in the full model.

Table 8: National Power's Generation Capacity (GW)

Zone	Name	National Power Must-run	National Power Variable	Industry Total
0	North [Western]	0.0	0.0	0.6
1	North [Eastern]	0.0	1.1	5.0
2	Yorkshire	0.7	6.0	11.4
3	N Wales and W Lancs	0.5	0.0	10.0
4	E Lancashire	0.0	0.0	0.0
5	Nottinghamshire	0.0	2.0	4.9
6	West Midlands	0.0	0.5	5.4
7	East Anglia	0.7	0.0	3.6
8	West and Wales	1.1	3.5	4.6
9	[Thames] Estuary	0.0	1.5	9.9
10	London [Inner and Outer]	0.0	0.9	2.6
12	South Coast	0.0	0.7	0.7
13	Wessex and Peninsula	0.0	0.0	1.9
		3.0	16.1	60.7

Source: NGC (1996) Figures do not sum to totals due to rounding

Prices are, unsurprisingly, higher and quantities lower. The bottom part of the table shows that the generators increase their profits by nearly £1 billion in each case – the profits coming from transmission are also higher, by £140 million. The biggest increase in profits comes with uniform pricing for demand but local pricing for generation. National Power has a strong incentive to create a constraint and raise prices in the South-West when demand is high, given its ownership of a high proportion of the local plant. This naturally feeds through into prices, but a uniform national price for demand implies that the demand reduction effect is spread across all generators, and National Power has a smaller proportion of the total generation than of the generation in the South-West. With optimal prices, the local demand response makes creating the constraint less profitable. In fact, it is profitable for National Power to create congestion in the South-West for the three highest demand levels in each season when there is a uniform price for demand, but only for the two highest levels with optimal pricing. Under the Pool system, National Power can still create a constraint without attracting a local demand response, but only gets the high price for the volume involved in counter-trading, rather than for all of its local output. The company does gain, however, from the opportunity of submitting very low bids for its capacity in the North, ensuring that all of it is first scheduled and then constrained off. With low bids, the company pays almost nothing to buy back its output. The net effect raises the company's profits, but is not enough to offset the lower volumes attracting high prices in the South-West. Consumer surplus falls by between £1.2 billion and £1.3 billion, and the overall impact of market power is to reduce

Table 9: The impact of market power

Pricing System	Optimal	Zonal (for Generators)	Uniform
Demand (TWh)	280.9	276.9	276.7
Losses (TWh)	1.53	1.63	1.64
Total Revenue (£m)	8846	8862	8947
Average Revenue (£/MWh)	31.49	32.01	32.33
Cost of counter-trading (£m)			383
Coefficient of Variation of prices:			
Within hours	0.30	0.49	0.00
Across and within hours	0.63	0.91	0.52
Changes relative to optimal pricing:			
Welfare (£m)		-107	-163
(% of revenue)		-1.2%	-1.8%
Consumer surplus (£m)		-90	-204
(% of revenue)		-1.0%	-2.3%
Generator profit (£m)		-16	42
(% of revenue)		-0.2%	0.5%
Cost of generation (£m)		32	59
(% of revenue)		0.4%	0.7%
Changes relative to marginal cost bidding:			
Welfare (£m)	-143	-185	-187
(% of revenue)	-1.8%	-2.3%	-2.4%
Consumer surplus (£m)	-1214	-1340	-1212
(% of revenue)	-15.3%	-16.9%	-15.3%
Generator profit (£m)	928	1014	884
(% of revenue)	11.7%	12.8%	11.1%
National Power's profit (£m)	147	310	270

welfare by between £143 million and £187 million, depending on the pricing rule. The impact is lowest under optimal pricing.

When we look at the choice of pricing rule, taking the presence of market power as given, the previous welfare ranking is unchanged, and the cost of choosing a sub-optimal rule increased. (These are the results in the centre of the table.) A uniform price for demand alone reduces welfare by £107 million, or 1.3% of the revenues with optimal pricing; two uniform prices reduce welfare by £163 million (1.8% of revenues). Consumers are now worse off with either of the sub-optimal pricing rules, although the Pool system continues to produce the worst result.[11] The choice of pricing rule now matters less to generators –

[11] Since the Pool was frequently criticised as being bad for consumers before it was abolished, it might be worth pointing out that this result is not due to the Pool's rules for pricing energy (since all of the pricing rules here

counter-trading was better for them with marginal cost pricing, but offers less scope for raising their profits under market power, and the two effects nearly cancel out. The reverse applies for a uniform demand price and locational generation prices, although generators are still slightly worse off than with optimal pricing.

VII. Conclusions

This paper has illustrated the benefits of applying optimal spot prices in a simple model of an electricity network based on England and Wales in 1996/97. Looking at operating costs alone, and taking nodal spot pricing as the optimum, welfare would fall by 0.8% of wholesale revenues if a uniform demand price were applied, and by 1.5% if uniform prices for both demand and generation were adopted. In the presence of market power, the cost of sub-optimal pricing rises to 1.2% and 1.8% of revenues, respectively. These may seem like small numbers, but they can be compared to Newbery and Pollitt's (1997) estimate of the net benefits from privatising the Central Electricity Generating Board, which was equal to 5% of the Board's costs. "Throwing away" up to one-third of the benefits of that exercise might sound rather more significant.

It must be remembered, however, that introducing optimal prices will often involve transfers between agents that are much greater than the net welfare gain. That is likely to make them hard to introduce from a political point of view. The attempt to introduce transmission loss factors in Great Britain, which would reduce prices in the north, and increase them in the south, is a good example of this. The resulting prices would provide better economic signals than the present system, and the issue was identified as one for future review when the market was first restructured, in 1990. Companies which stood to lose from the change used the industry's processes (including two unsuccessful appeals to the regulator) and legal action to stop it. Changes to the market's governance when the New Electricity Trading Arrangements were introduced allowed the regulator to press ahead with the change in England and Wales, but when the government extended the market to Scotland, it over-ruled the regulator and decreed that prices would continue to be uniform.

The gains from optimal prices will be greater once we consider their effect as investment signals to generators.[12] In the short term, moving towards optimal prices increases the output from stations in "good" locations, but since all the stations with low operating costs would have been dispatched under uniform pricing in any case, the stations where output rises must have relatively high operating costs, which limits the gain. Consider a long, heavily loaded line with a station at each end, and marginal transmission losses of 10%. If the marginal costs of the station at the "right" end of the line are 5% higher than at the other station, the saving from reallocating generation between the stations is only 5%.

---

are based on the marginal unit setting the price) but on the way transmission is handled, through counter-trading.

[12] Spot prices are not the only way to send investment signals, however. In England and Wales, the National Grid Company has regionally differentiated capacity-based charges, encouraging investment in the south.

When considering investment, however, we can save the full 10% by placing the new station at the right end of the line, and the saving applies to capital as well as operating costs.

We have assumed that the system can be successfully dispatched without the use of spot prices, and thus ignored the issue of bypass. When prices are not equal to marginal costs, some agents may wish to leave the main market and deal independently, reducing its efficiency. Hogan (1998) describes a failed experiment in the Pennsylvania-New Jersey-Maryland (PJM) Interconnection in 1997. Customers faced a price based on an unconstrained system, but transmission constraints meant that some generators were forced not to run. They received no compensation for this, and had a strong incentive to arrange a bilateral transaction with a customer at a price between their marginal cost and the unconstrained price. Once these generators had done so, their output could not be reduced, and the system operator had to constrain some other generators instead. These then had an incentive to act in the same way, and the system operator was forced to ban bilateral trading before it ran out of dispatchable generation. A revised market mechanism uses spot prices, and is not threatened by bilateral trading. Hogan argues that any move to use zonal prices instead of nodal spot prices would create similar perverse incentives if used as a means of system co-ordination, and would be unlikely to simplify the calculation or interpretation of those prices.

One way to get round the problem of bypass is to ban bilateral trading from the start, as was done in England and Wales under the Pool system. Another is to arrange compensation payments for constrained generators, funded by all consumers, so that no-one has an incentive to leave the market, but this reduces the incentive to avoid congested locations. There are ways to make an electricity market work without nodal spot pricing.

The important question is whether nodal spot pricing would make the market work better. This paper estimates that moving from uniform pricing to nodal pricing could raise welfare by 1.5% of the generators' revenues in a competitive market, and would be less vulnerable to the exercise of market power than alternative systems. They would also improve the investment signals sent to generators. These are gains worth pursuing.

Bibliography

Bialek, J., M.Hartley and S.Topping (2003) "Average Zonal Transmission" *IEE Power Engineer*, October/November, pp 34-37

Borenstein, S., J. Bushnell and S. Stoft, (2000) "The Competitive Effects of Transmission Capacity in a Deregulated Electricity Industry", *RAND Journal of Economics*, vol. 31 no. 2, pp. 294-325

Cardell, J.B., C. C. Hitt and W. W. Hogan (1997) "Market Power and Strategic Interaction in Electricity Networks" *Resource and Energy Economics* Vol. 19, No. 1-2, pp. 109-137

Gilbert, R., K. Neuhoff, and D. Newbery (2004) "Allocating transmission to mitigate market power in electricity networks" *RAND Journal of Economics* vol 35 no 4

Green, R.J. (1994) *Do Electricity Transmission Networks need Optimal Spot Prices?* Mimeo, University of Cambridge

Green, R.J. (1997) "Electricity Transmission Pricing - An International Comparison" *Utilities Policy*, Vol. 6, No 3, pp. 177-184

Green, R.J. (1998) *Electricity Transmission Pricing: How much does it cost to get it wrong?* Power Working Paper PWP-058, University of California Energy Institute

Hogan, W.W. (1992) "Contract Networks for Electric Power Transmission" *Journal of Regulatory Economics,* Vol. 4, No 3, pp. 211-242

Hogan, W.W. (1998) *Independent System Operator: Pricing and Flexibility in a Competitive Electricity Market*, mimeo, Harvard University

Hsu, M. (1997) "An Introduction to the Pricing of Electric Power Transmission" *Utilities Policy*, Vol. 6, No 3, pp. 257-270

Ilic, M.D., Y.T.Yoon, A.Zobian and M.E.Paravalos (1997) "Toward Regional Transmission Provision and its Pricing in New England" *Utilities Policy*, Vol. 6, No. 3, pp. 245-256

Joskow, P.L. and J.Tirole (2000) "Transmission Rights and Market Power on Electric Power Networks", *RAND Journal of Economics*, vol. 31, no. 3, Autumn, pp. 450-87

Macatangay, R.E. (1997) *Space-time Prices of Wholesale Electricity in England and Wales* mimeo, University of Manchester

Newbery, D.M. and M.G.Pollitt (1997) "The Restructuring and Privatisation of the CEGB: Was It Worth It?" *Journal of Industrial Economics* Vol. XLV, No. 3, pp. 269-303

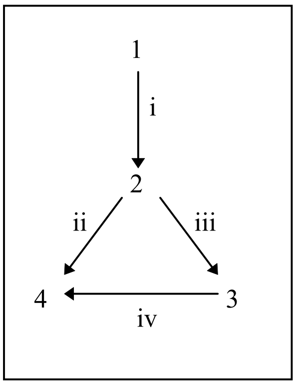
Oren, S.S., (1997) "Economic inefficiency of passive transmission rights in congested electricity systems with competitive generation" *Energy Journal,* vol. 18 no. 1, pp 63-83

Schweppe, F.C., M.C.Caramanis, R.D.Tabors and R.E.Bohn (1988) *Spot Pricing of Electricity*, Norwell, Ma: Kluwer

Appendix: The DC Load Flow Model.

The DC Load flow equations are a simple planning tool which may be used to provide an approximate relationship between generation and demand at each point on a network, and the power flows along each line. They depend upon a number of approximations which make them inadequate for detailed system planning, but should be adequate to illustrate the possibilities of unit transmission charges. The full equations are derived in appendix D of Schweppe et al, (1988); this appendix presents only their most simple form.

The network consists of N nodes which are linked by L lines. The net power injection (generation minus demand) at node j is denoted by y_j. The sum of the net injections must equal the transmission losses (l) on the system, under the energy balance constraint. In the DC equations, this means that one of the nodes is not explicitly modelled, and its net injection is effectively treated as a residual, determined by this constraint. The other N-1 injections form a vector $\underline{y}$. The flow along line i is denoted by z_i, and $\underline{z}$ is the N-vector of these flows. The network incidence matrix, Ã, shows which buses are connected by each line. A simple example is given below, in which all power is assumed to flow from the lower to the higher numbered node. The node from which power flows is given an entry of +1, and the node to which power flows is given an entry of -1. If the power actually flows in the opposite direction to that assumed, then z_i will be negative. Because Ã is singular, we remove the column corresponding to the swing bus to create a non-singular N * (N - 1) matrix A. In this example, node number 2 is the swing bus, and so we delete the second column of Ã to create A.



$$\tilde{A} \quad \text{node} \qquad \textit{line} \begin{array}{c} \\ i \\ ii \\ iii \\ iv \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix} \end{array}$$

$$A \quad \text{node} \qquad \textit{line} \begin{array}{c} \\ i \\ ii \\ iii \\ iv \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \end{array}$$

R is a $N \times N$ diagonal matrix, with the resistance of line i as the ii-th element, and zeros elsewhere. The three DC load flow equations are:

$$\underline{z} = R^{-1} A \left(A^T R^{-1} A \right)^{-1} \underline{y} \qquad \text{(line flow equation)}$$

$$l = \underline{z}^T R \underline{z} \qquad \text{(transmission losses)}$$

$$\sum_{j=1}^{N} y_j - l = 0 \qquad \text{(energy balance constraint)}$$

The energy balance constraint has been written to ensure that a given set of N net injections are consistent. An alternative is to rewrite the equation with the injection at the swing bus on the left hand side, solving for this as the residual from N-1 predetermined injections. This effectively assumes that the marginal generator will be located at the swing bus. In practice, the merit order determines the location of the marginal generator, which is unlikely to be at a pre-defined swing bus, and the energy balance constraint as written here is used to ensure that its output (a choice variable) is consistent with electrical equilibrium.